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Distribution System Modeling and Analysis

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21.1 Modeling

Radial distribution feeders are characterized by having only one path for power to flow from the source (distribution substation) to each customer. A typical distribution system will consist of one or more distribution substations consisting of one or more “feeders.” Components of the feeder may consist of the following:

- Three-phase primary “main” feeder
- Three-phase, two-phase (“V” phase), and single-phase laterals
- Step-type voltage regulators or load tap changing transformer (LTC)
- In-line transformers
- Shunt capacitor banks
- Three-phase, two-phase, and single-phase loads
- Distribution transformers (step-down to customer’s voltage)

The loading of a distribution feeder is inherently unbalanced because of the large number of unequal single-phase loads that must be served. An additional unbalance is introduced by the nonequilateral conductor spacings of the three-phase overhead and underground line segments.

Because of the nature of the distribution system, conventional power-flow and short-circuit programs used for transmission system studies are not adequate. Such programs display poor convergence characteristics for radial systems. The programs also assume a perfectly balanced system so that a single-phase equivalent system is used.

If a distribution engineer is to be able to perform accurate power-flow and short-circuit studies, it is imperative that the distribution feeder be modeled as accurately as possible. This means that three-phase models of the major components must be utilized. Three-phase models for the major components will be developed in the following sections. The models will be developed in the “phase frame” rather than applying the method of symmetrical components.

Figure 21.1 shows a simple one-line diagram of a three-phase feeder; it illustrates the major components of a distribution system. The connecting points of the components will be referred to as “nodes.” Note in the figure that the phasing of the line segments is shown. This is important if the most accurate models are to be developed.

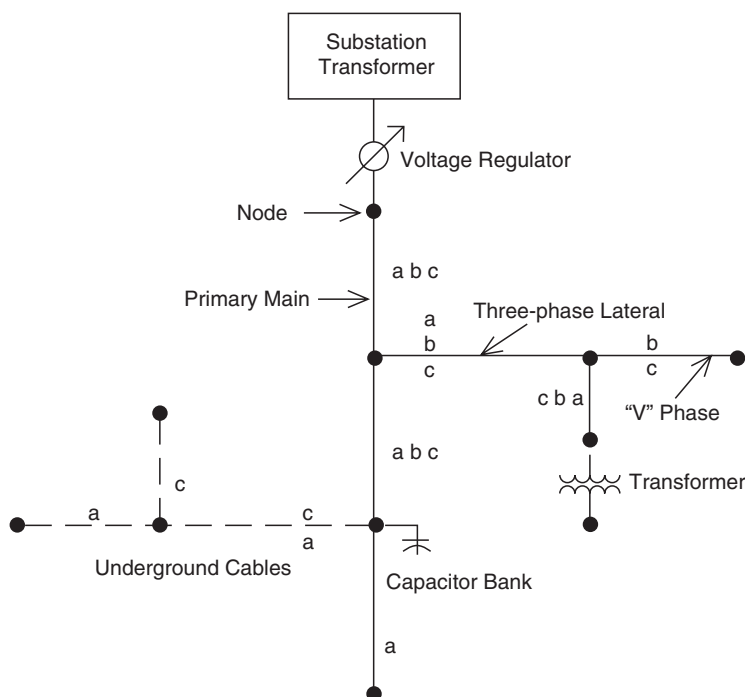


FIGURE 21.1 Distribution feeder.

The following sections will present generalized three-phase models for the “series” components of a feeder (line segments, voltage regulators, transformer banks). Additionally, models are presented for the “shunt” components (loads, capacitor banks). Finally, the “ladder iterative technique” for power-flow studies using the models is presented along with a method for computing short-circuit currents for all types of faults.

21.1.1 Line Impedance

The determination of the impedances for overhead and underground lines is a critical step before analysis of the distribution feeder can begin. Depending upon the degree of accuracy required, impedances can be calculated using Carson’s equations where no assumptions are made, or the impedances can be determined from tables where a wide variety of assumptions are made. Between these two limits are other techniques, each with their own set of assumptions.

21.1.1.1 Carson’s Equations

Since a distribution feeder is inherently unbalanced, the most accurate analysis should not make any assumptions regarding the spacing between conductors, conductor sizes, or transposition. In a classic paper, John Carson developed a technique in 1926 whereby the self and mutual impedances for n overhead conductors can be determined. The equations can also be applied to underground cables. In 1926, this technique was not met with a lot of enthusiasm because of the tedious calculations that had to be done on the slide rule and by hand. With the advent of the digital computer, Carson’s equations have now become widely used.

In his paper, Carson assumes the earth is an infinite, uniform solid, with a flat uniform upper surface and a constant resistivity. Any “end effects” introduced at the neutral grounding points are not large at power frequencies, and therefore are neglected. The original Carson equations are given in [Eqs. \(21.1\)](#) and [\(21.2\)](#).

Self-impedance:

$$\hat{z}_{ii} = r_i + 4\varpi P_{ii}G + j \left(X_i + 2\varpi G_{ii} \times \ln \frac{S_{ii}}{R_i} + 4\varpi Q_{ii}G \right) \Omega/\text{mile} \quad (21.1)$$

Mutual impedance:

$$\hat{z}_{ij} = 4\varpi P_{ij}G + j \left(2\varpi G \ln \frac{S_{ij}}{D_{ij}} + 4\varpi Q_{ij}G \right) \Omega/\text{mile} \quad (21.2)$$

where $\hat{z}_{ii}$ = self-impedance of conductor i in Ω/mile
 $\hat{z}_{ij}$ = mutual impedance between conductors i and j in Ω/mile
 r_i = resistance of conductor i in Ω/mile
 ν = system angular frequency in radians per second
 G = $0.1609347 \times 10^{-7} \Omega\text{cm}/\text{abohm-mile}$
 R_i = radius of conductor i in feet
 GMR_i = geometric mean radius of conductor i in feet
 f = system frequency in Hertz
 r = resistivity of earth in Ωm
 D_{ij} = distance between conductors i and j in feet
 S_{ij} = distance between conductor i and image j in feet
 q_{ij} = angle between a pair of lines drawn from conductor i to its own image and to the image of conductor j

$$X_i = 2\omega G \ln \frac{R_i}{\text{GMR}_i} \Omega/\text{mile} \quad (21.3)$$

$$P_{ij} = \frac{\pi}{8} - \frac{1}{3\sqrt{2}} k_{ij} \cos(\theta_{ij}) + \frac{k_{ij}^2}{16} \cos(2\theta_{ij}) \left(0.6728 + \ln \frac{2}{k_{ij}} \right) \quad (21.4)$$

$$Q_{ij} = -0.0386 + \frac{1}{2} \ln \frac{2}{k_{ij}} + \frac{1}{3\sqrt{2}} k_{ij} \cos(\theta_{ij}) \quad (21.5)$$

$$k_{ij} = 8.565 \times 10^{-4} \times S_{ij} \times \sqrt{\frac{f}{\rho}} \quad (21.6)$$

As indicated above, Carson made use of conductor images; that is, every conductor at a given distance above ground has an image conductor the same distance below ground. This is illustrated in [Fig. 21.2](#).

21.1.1.2 Modified Carson's Equations

Only two approximations are made in deriving the “modified Carson equations.” These approximations involve the terms associated with P_{ij} and Q_{ij} . The approximations are shown below:

$$P_{ij} = \frac{\pi}{8} \quad (21.7)$$

$$Q_{ij} = -0.03860 + \frac{1}{2} \ln \frac{2}{k_{ij}} \quad (21.8)$$

It is also assumed

f = frequency = 60 Hertz

r = resistivity = 100 Ωm

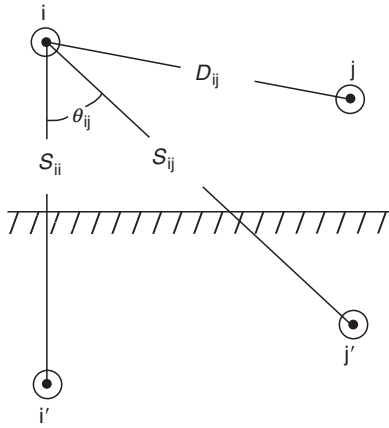


FIGURE 21.2 Conductors and images.

ing matrix will be 6×6 . The primitive impedance matrix for a three-phase line consisting of m neutrals will be of the form

$$[z_{\text{primitive}}] = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & | & \hat{z}_{an1} & \cdot & \hat{z}_{anm} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & | & \hat{z}_{bn1} & \cdot & \hat{z}_{bnm} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & | & \hat{z}_{cn1} & \cdot & \hat{z}_{cnm} \\ \hline \hat{z}_{n1a} & \hat{z}_{n1b} & \hat{z}_{n1c} & | & \hat{z}_{n1n1} & \cdot & \hat{z}_{n1nm} \\ \cdot & \cdot & \cdot & | & \cdot & \cdot & \cdot \\ \hat{z}_{nma} & \hat{z}_{nmb} & \hat{z}_{nmc} & | & \hat{z}_{nmn1} & \cdot & \hat{z}_{nmnm} \end{bmatrix} \quad (21.11)$$

In partitioned form Eq. (20.11) becomes

$$[z_{\text{primitive}}] = \begin{bmatrix} [\hat{z}_{ij}] & [\hat{z}_{in}] \\ [\hat{z}_{nj}] & [\hat{z}_{nn}] \end{bmatrix} \quad (21.12)$$

21.1.1.4 Phase Impedance Matrix

For most applications, the primitive impedance matrix needs to be reduced to a 3×3 phase frame matrix consisting of the self and mutual equivalent impedances for the three phases. One standard method of reduction is the “Kron” reduction (1952) where the assumption is made that the line has a multigrounded neutral. The Kron reduction results in the “phase impedances matrix” determined by using Eq. (21.13) below:

$$[z_{abc}] = [\hat{z}_{ij}] - [\hat{z}_{in}][\hat{z}_{nn}]^{-1}[\hat{z}_{nj}] \quad (21.13)$$

It should be noted that the phase impedance matrix will always be of rotation a–b–c no matter how the phases appear on the pole. That means that always row and column 1 in the matrix will represent phase a, row and column 2 will represent phase b, row and column 3 will represent phase c.

For two-phase (V-phase) and single-phase lines in grounded wye systems, the modified Carson equations can be applied, which will lead to initial 3×3 and 2×2 primitive impedance matrices. Kron reduction will reduce the matrices to 2×2 and a single element. These matrices can be expanded to 3×3 phase frame matrices by the addition of rows and columns consisting of zero elements for the missing phases. The phase frame matrix for a three-wire delta line is determined by the application of Carson’s equations without the Kron reduction step.

Using these approximations and assumptions, Carson’s equations reduce to:

$$\hat{z}_{ii} = r_i + 0.0953 + j0.12134 \left(\ln \frac{1}{\text{GMR}_i} + 7.93402 \right) \Omega/\text{mile} \quad (21.9)$$

$$\hat{z}_{ij} = 0.0953 + j0.12134 \left(\ln \frac{1}{D_{ij}} + 7.93402 \right) \Omega/\text{mile} \quad (21.10)$$

21.1.1.3 Overhead and Underground Lines

Equations (21.9) and (21.10) can be used to compute an $n_{\text{cond}} \times n_{\text{cond}}$ “primitive impedance” matrix. For an overhead four wire, grounded wye distribution line segment, this will result in a 4×4 matrix. For an underground grounded wye line segment consisting of three concentric neutral cables, the result-

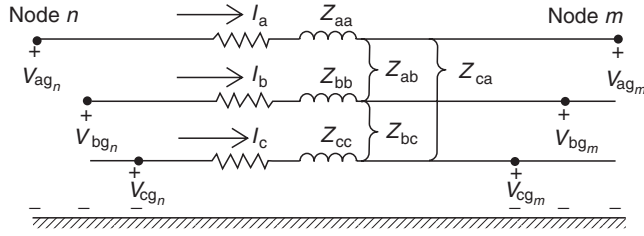


FIGURE 21.3 Three-phase line segment.

The phase frame matrix can be used to accurately determine the voltage drops on the feeder line segments once the currents flowing have been determined. Since no approximations (transposition, for example) have been made regarding the spacing between conductors, the effect of the mutual coupling between phases is accurately taken into account. The application of Carson's equations and the phase frame matrix leads to the most accurate model of a line segment. Figure 21.3 shows the equivalent circuit of a line segment.

The voltage equation in matrix form for the line segment is given by the following equation:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (21.14)$$

where $Z_{ij} = z_{ij} \times \text{length}$

The phase impedance matrix is defined in Eq. (21.15). The phase impedance matrix for single-phase and V-phase lines will have a row and column of zeros for each missing phase

$$[Z_{abc}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \quad (21.15)$$

Equation (21.14) can be written in condensed form as

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}][I_{abc}] \quad (21.16)$$

This condensed notation will be used throughout the document.

21.1.1.5 Sequence Impedances

Many times the analysis of a feeder will use the positive and zero sequence impedances for the line segments. There are basically two methods for obtaining these impedances. The first method incorporates the application of Carson's equations and the Kron reduction to obtain the phase frame impedance matrix. The 3×3 "sequence impedance matrix" can be obtained by

$$[z_{012}] = [A_s]^{-1} [z_{abc}] [A_s] \quad \Omega/\text{mile} \quad (21.17)$$

where

$$[A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \quad (21.18)$$

$$a_s = 1.0 \angle 120 \quad a_s^2 = 1.0 \angle 240$$

The resulting sequence impedance matrix is of the form:

$$[z_{012}] = \begin{bmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{bmatrix} \Omega/\text{mile} \quad (21.19)$$

where z_{00} = the zero sequence impedance

z_{11} = the positive sequence impedance

z_{22} = the negative sequence impedance

In the idealized state, the off-diagonal terms of Eq. (21.19) would be zero. When the off-diagonal terms of the phase impedance matrix are all equal, the off-diagonal terms of the sequence impedance matrix will be zero. For high-voltage transmission lines, this will generally be the case because these lines are transposed, which causes the mutual coupling between phases (off-diagonal terms) to be equal. Distribution lines are rarely if ever transposed. This causes unequal mutual coupling between phases, which causes the off-diagonal terms of the phase impedance matrix to be unequal. For the nontransposed line, the diagonal terms of the phase impedance matrix will also be unequal. In most cases, the off-diagonal terms of the sequence impedance matrix are very small compared to the diagonal terms and errors made by ignoring the off-diagonal terms are small.

Sometimes the phase impedance matrix is modified such that the three diagonal terms are equal and all of the off-diagonal terms are equal. The usual procedure is to set the three diagonal terms of the phase impedance matrix equal to the average of the diagonal terms of Eq. (21.15) and the off-diagonal terms equal to the average of the off-diagonal terms of Eq. (21.15). When this is done, the self and mutual impedances are defined as

$$z_s = \frac{1}{3}(z_{aa} + z_{bb} + z_{cc}) \quad (21.20)$$

$$z_m = \frac{1}{3}(z_{ab} + z_{bc} + z_{ca}) \quad (21.21)$$

The phase impedance matrix is now defined as

$$[z_{abc}] = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} \quad (21.22)$$

When Eq. (21.17) is used with this phase impedance matrix, the resulting sequence matrix is diagonal (off-diagonal terms are zero). The sequence impedances can be determined directly as

$$\begin{aligned} z_{00} &= z_s + 2z_m \\ z_{11} &= z_{22} = z_s - z_m \end{aligned} \quad (21.23)$$

A second method that is commonly used to determine the sequence impedances directly is to employ the concept of geometric mean distances (GMDs). The GMD between phases is defined as

$$D_{ij} = \text{GMD}_{ij} = \sqrt[3]{D_{ab}D_{bc}D_{ca}} \quad (21.24)$$

The GMD between phases and neutral is defined as

$$D_{in} = \text{GMD}_{in} = \sqrt[3]{D_{an}D_{bn}D_{cn}} \quad (21.25)$$

The GMDs as defined above are used in Eqs. (21.9) and (21.10) to determine the various self and mutual impedances of the line resulting in

$$\hat{z}_{ii} = r_i + 0.0953 + j0.12134 \left[\ln \left(\frac{1}{\text{GMR}_i} \right) + 7.93402 \right] \quad (21.26)$$

$$\hat{z}_{nn} = r_n + 0.0953 + j0.12134 \left[\ln \left(\frac{1}{\text{GMR}_n} \right) + 7.93402 \right] \quad (21.27)$$

$$\hat{z}_{ij} = 0.0953 + j0.12134 \left[\ln \left(\frac{1}{D_{ij}} \right) + 7.93402 \right] \quad (21.28)$$

$$\hat{z}_{in} = 0.0953 + j0.12134 \left[\ln \left(\frac{1}{D_{in}} \right) + 7.93402 \right] \quad (21.29)$$

Equations (21.26) through (21.29) will define a matrix of order $\text{ncond} \times \text{ncond}$, where ncond is the number of conductors (phases plus neutrals) in the line segment. Application of the Kron reduction [Eq. (21.13)] and the sequence impedance transformation [Eq. (21.23)] lead to the following expressions for the zero, positive, and negative sequence impedances:

$$z_{00} = \hat{z}_{ii} + 2\hat{z}_{ij} - 3 \left(\frac{\hat{z}_{in}^2}{\hat{z}_{nn}} \right) \Omega/\text{mile} \quad (21.30)$$

$$\begin{aligned} z_{11} &= z_{22} = \hat{z}_{ii} - \hat{z}_{ij} \\ z_{11} &= z_{22} = r_i + j0.12134 \times \ln \left(\frac{D_{ij}}{\text{GMR}_i} \right) \Omega/\text{mile} \end{aligned} \quad (21.31)$$

Equation (21.31) is recognized as the standard equation for the calculation of the line impedances when a balanced three-phase system and transposition are assumed.

Example 21.1

The spacings for an overhead three-phase distribution line are constructed as shown in Fig. 21.4. The phase conductors are 336,400 26/7 ACSR (Linnet) and the neutral conductor is 4/0 6/1 ACSR.

- Determine the phase impedance matrix.
- Determine the positive and zero sequence impedances.

Solution

From the table of standard conductor data, it is found that

336,400 26/7 ACSR: $\text{GMR} = 0.0244 \text{ ft}$

Resistance = $0.306 \Omega/\text{mile}$

4/0 6/1 ACSR: $\text{GMR} = 0.00814 \text{ ft}$

Resistance = $0.5920 \Omega/\text{mile}$

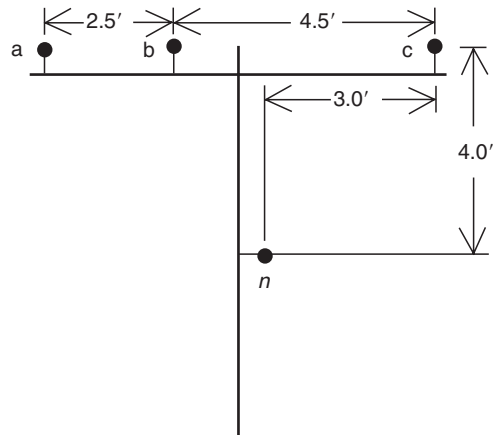


FIGURE 21.4 Three-phase distribution line spacings.

From Fig. 21.4 the following distances between conductors can be determined:

$$\begin{aligned} D_{ab} &= 2.5 \text{ ft} & D_{bc} &= 4.5 \text{ ft} & D_{ca} &= 7.0 \text{ ft} \\ D_{an} &= 5.6569 \text{ ft} & D_{bn} &= 4.272 \text{ ft} & D_{cn} &= 5.0 \text{ ft} \end{aligned}$$

Applying Carson's modified equations [Eqs. (21.9) and (21.10)] results in the primitive impedance matrix.

$$[\hat{z}] = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\ 0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.6873 + j1.5465 \end{bmatrix} \quad (21.32)$$

The Kron reduction of Eq. (21.13) results in the phase impedance matrix

$$[z_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile} \quad (21.33)$$

The phase impedance matrix of Eq. (21.33) can be transformed into the sequence impedance matrix with the application of Eq. (21.17)

$$[z_{012}] = \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & -0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & -0.0723 - j0.0060 \\ 0.0256 + j0.0115 & 0.0723 - j0.0059 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile} \quad (21.34)$$

In Eq. (21.34), the 1,1 term is the zero sequence impedance, the 2,2 term is the positive sequence impedance, and the 3,3 term is the negative sequence impedance. Note that the off-diagonal terms are not zero, which implies that there is mutual coupling between sequences. This is a result of the nonsymmetrical spacing between phases. With the off-diagonal terms nonzero, the three sequence networks representing the line will not be independent. However, it is noted that the off-diagonal terms are small relative to the diagonal terms.

In high-voltage transmission lines, it is usually assumed that the lines are transposed and that the phase currents represent a balanced three-phase set. The transposition can be simulated in this example by replacing the diagonal terms of Eq. (21.33) with the average value of the diagonal terms ($0.4619 + j1.0638$) and replacing each off-diagonal term with the average of the off-diagonal terms ($0.1558 + j0.4368$). This modified phase impedance matrix becomes

$$[z_{1abc}] = \begin{bmatrix} 0.3619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.3619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.3619 + j1.0638 \end{bmatrix} \Omega/\text{mile} \quad (21.35)$$

Using this modified phase impedance matrix in the symmetrical component transformation, Eq. (21.17) results in the modified sequence impedance matrix

$$[z_{1012}] = \begin{bmatrix} 0.7735 + j1.9373 & 0 & 0 \\ 0 & 0.3061 + j0.6270 & 0 \\ 0 & 0 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile} \quad (21.36)$$

Note now that the off-diagonal terms are all equal to zero, meaning that there is no mutual coupling between sequence networks. It should also be noted that the zero, positive, and negative sequence impedances of Eq. (21.36) are exactly equal to the same sequence impedances of Eq. (21.34).

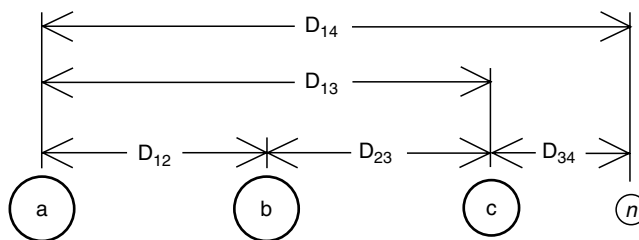


FIGURE 21.5 Three-phase underground with additional neutral.

The results of this example should not be interpreted to mean that a three-phase distribution line can be assumed to have been transposed. The original phase impedance matrix of Eq. (21.33) must be used if the correct effect of the mutual coupling between phases is to be modeled.

21.1.1.6 Underground Lines

Figure 21.5 shows the general configuration of three underground cables (concentric neutral, or tape shielded) with an additional neutral conductor.

Carson's equations can be applied to underground cables in much the same manner as for overhead lines. The circuit of Fig. 21.5 will result in a 7×7 primitive impedance matrix. For underground circuits that do not have the additional neutral conductor, the primitive impedance matrix will be 6×6 .

Two popular types of underground cables in use today are the "concentric neutral cable" and the "tape shield cable." To apply Carson's equations, the resistance and GMR of the phase conductor and the equivalent neutral must be known.

21.1.1.7 Concentric Neutral Cable

Figure 21.6 shows a simple detail of a concentric neutral cable. The cable consists of a central phase conductor covered by a thin layer of nonmetallic semiconducting screen to which is bonded the insulating material. The insulation is then covered by a semiconducting insulation screen. The solid strands of concentric neutral are spiralled around the semiconducting screen with a uniform spacing between strands. Some cables will also have an insulating "jacket" encircling the neutral strands.

In order to apply Carson's equations to this cable, the following data needs to be extracted from a table of underground cables:

- d_c = phase conductor diameter (in.)
- d_{od} = nominal outside diameter of the cable (in.)
- d_s = diameter of a concentric neutral strand (in.)
- GMR_c = geometric mean radius of the phase conductor (ft)

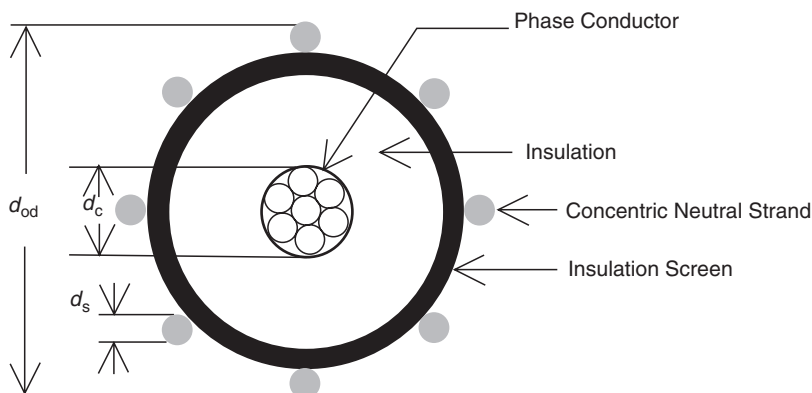


FIGURE 21.6 Concentric neutral cable.

GMR_s = geometric mean radius of a neutral strand (ft)
 r_c = resistance of the phase conductor (Ω /mile)
 r_s = resistance of a solid neutral strand (Ω /mile)
 k = number of concentric neutral strands

The geometric mean radii of the phase conductor and a neutral strand are obtained from a standard table of conductor data. The equivalent geometric mean radius of the concentric neutral is given by

$$GMR_{cn} = \sqrt[k]{GMR_s \times kR^{k-1}} \text{ ft} \quad (21.37)$$

where R = radius of a circle passing through the center of the concentric neutral strands

$$R = \frac{d_{od} - d_s}{24} \text{ ft} \quad (21.38)$$

The equivalent resistance of the concentric neutral is

$$r_{cn} = \frac{r_s}{k} \Omega/\text{mile} \quad (21.39)$$

The various spacings between a concentric neutral and the phase conductors and other concentric neutrals are as follows:

Concentric neutral to its own phase conductor

$$D_{ij} = R[\text{Eq. (21.38) above}]$$

Concentric neutral to an adjacent concentric neutral

$$D_{ij} = \text{center-to-center distance of the phase conductors}$$

Concentric neutral to an adjacent phase conductor

Figure 21.7 shows the relationship between the distance between centers of concentric neutral cables and the radius of a circle passing through the centers of the neutral strands.

The GMD between a concentric neutral and an adjacent phase conductor is given by the following equation:

$$D_{ij} = \sqrt[k]{D_{nm}^k - R^k} \text{ ft} \quad (21.40)$$

where D_{nm} = center-to-center distance between phase conductors

For cables buried in a trench, the distance between cables will be much greater than the radius R and therefore very little error is made if D_{ij} in Eq. (21.40) is set equal to D_{nm} . For cables in conduit, that assumption is not valid.

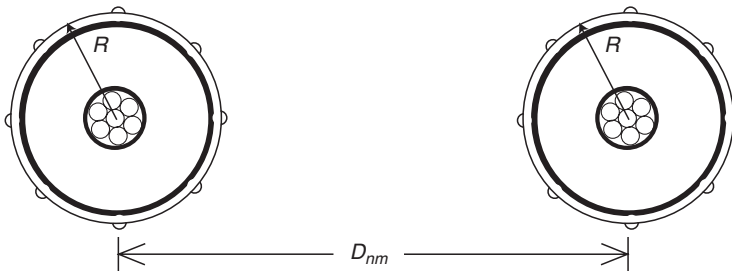


FIGURE 21.7 Distances between concentric neutral cables.

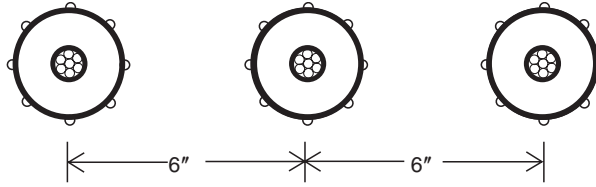


FIGURE 21.8 Three-phase concentric neutral cable spacing.

Example 21.2

Three concentric neutral cables are buried in a trench with spacings as shown in Fig. 21.8. The cables are 15 kV, 250,000 CM stranded all aluminum with 13 strands of #14 annealed coated copper wires (1/3 neutral). The data for the phase conductor and neutral strands from a conductor data table are

250,000 AA phase conductor: $GMR_p = 0.0171$ ft, resistance = $0.4100 \Omega/\text{mile}$
 #14 copper neutral strands: $GMR_s = 0.00208$ ft, resistance = $14.87 \Omega/\text{mile}$
 Diameter (d_s) = 0.0641 in.

The equivalent GMR of the concentric neutral [Eq. (21.37)] = 0.04864 ft

The radius of the circle passing through strands [Eq. (21.38)] = 0.0511 ft

The equivalent resistance of the concentric neutral [Eq. (21.39)] = $1.1440 \Omega/\text{mile}$

Since R (0.0511 ft) is much less than D_{12} (0.5 ft) and D_{13} (1.0 ft), then the distances between concentric neutrals and adjacent phase conductors are the center-to-center distances of the cables.

Applying Carson's equations results in a 6×6 primitive impedance matrix. This matrix in partitioned form [Eq. (21.12)] is:

$$\begin{aligned}
 [z_{ij}] &= \begin{bmatrix} 0.5053 + j1.4564 & 0.0953 + j1.0468 & 0.0953 + j0.9627 \\ 0.0953 + j1.0468 & 0.5053 + j1.4564 & 0.0953 + j1.0468 \\ 0.0953 + j0.9627 & 0.0953 + j1.0468 & 0.5053 + j1.4564 \end{bmatrix} \\
 [z_{in}] &= \begin{bmatrix} 0.0953 + j1.3236 & 0.0953 + j1.0468 & 0.0953 + j0.9627 \\ 0.0953 + j1.0468 & 0.0953 + j1.3236 & 0.0953 + j1.0468 \\ 0.0953 + j0.9627 & 0.0953 + j1.0468 & 0.0953 + j1.3236 \end{bmatrix} \\
 [z_{nj}] &= [z_{in}] \\
 [z_{mn}] &= \begin{bmatrix} 1.2393 + j1.3296 & 0.0953 + j1.0468 & 0.0953 + j0.9627 \\ 0.0953 + j1.0468 & 1.2393 + j1.3296 & 0.0953 + j1.0468 \\ 0.0953 + j0.9627 & 0.0953 + j1.0468 & 1.2393 + j1.3296 \end{bmatrix}
 \end{aligned}$$

Using the Kron reduction [Eq. (21.13)] results in the phase impedance matrix

$$[z_{abc}] = \begin{bmatrix} 0.7982 + j0.4463 & 0.3192 + j0.0328 & 0.2849 - j0.0143 \\ 0.3192 + j0.0328 & 0.7891 + j0.4041 & 0.3192 + j0.0328 \\ 0.2849 - j0.0143 & 0.3192 + j0.0328 & 0.7982 + j0.4463 \end{bmatrix} \Omega/\text{mile}$$

The sequence impedance matrix for the concentric neutral three-phase line is determined using Eq. (21.3). The resulting sequence impedance matrix is

$$[z_{012}] = \begin{bmatrix} 1.4106 + j0.4665 & -0.0028 - j0.0081 & -0.0056 + j0.0065 \\ -0.0056 + j0.0065 & 0.4874 + j0.4151 & -0.0264 + j0.0451 \\ -0.0028 - j0.0081 & 0.0523 + j0.0003 & 0.4867 + j0.4151 \end{bmatrix} \Omega/\text{mile}$$

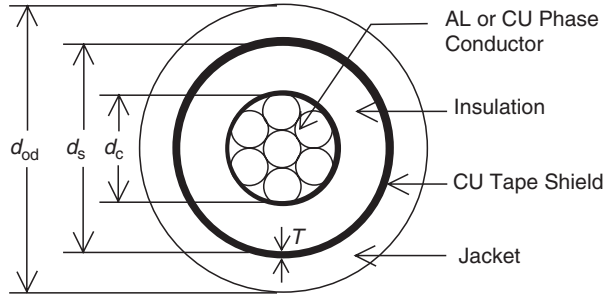


FIGURE 21.9 Taped shielded cable.

21.1.1.8 Tape Shielded Cables

Figure 21.9 shows a simple detail of a tape shielded cable.

Parameters of Fig. 21.9 are

- d_c = diameter of phase conductor (in.)
- d_s = inside diameter of tape shield (in.)
- d_{od} = outside diameter over jacket (in.)
- T = thickness of copper tape shield in mils
= 5 mils (standard)

Once again, Carson's equations will be applied to calculate the self-impedances of the phase conductor and the tape shield as well as the mutual impedance between the phase conductor and the tape shield.

The resistance and GMR of the phase conductor are found in a standard table of conductor data.

The resistance of the tape shield is given by

$$r_{\text{shield}} = \frac{18.826}{d_s T} \Omega/\text{mile} \quad (21.41)$$

The resistance of the tape shield given in Eq. (21.41) assumes a resistivity of 100 Ωm and a temperature of 50°C. The diameter of the tape shield d_s is given in inches and the thickness of the tape shield T is in mils.

The GMR of the tape shield is given by

$$\text{GMR}_{\text{shield}} = \frac{\frac{d_s}{2} - \frac{T}{2000}}{12} \text{ ft} \quad (21.42)$$

The various spacings between a tape shield and the conductors and other tape shields are as follows:

Tape shield to its own phase conductor

$$D_{ij} = \text{GMR}_{\text{tape}} = \text{radius to midpoint of the shield} \quad (21.43)$$

Tape shield to an adjacent tape shield

$$D_{ij} = \text{center-to-center distance of the phase conductors} \quad (21.44)$$

Tape shield to an adjacent phase or neutral conductor

$$D_{ij} = D_{nm} \quad (21.45)$$

where D_{nm} = center-to-center distance between phase conductors.

In applying Carson's equations for both concentric neutral and tape shielded cables, the numbering of conductors and neutrals is important. For example, a three-phase underground circuit with an additional neutral conductor must be numbered as

- 1 = phase conductor #1
- 2 = phase conductor #2
- 3 = phase conductor #3
- 4 = neutral of conductor #1
- 5 = neutral of conductor #2
- 6 = neutral of conductor #3
- 7 = additional neutral conductor (if present)

Example 21.3

A single-phase circuit consists of a 1/0 AA tape shielded cable and a 1/0 CU neutral conductor as shown in Fig. 21.10.

Cable Data: 1/0 AA

- Inside diameter of tape shield = $d_s = 1.084$ in.
- Resistance = $0.97 \Omega/\text{mile}$
- GMR_p = 0.0111 ft
- Tape shield thickness = $T = 8$ mils

Neutral Data: 1/0 Copper, 7 strand

- Resistance = $0.607 \Omega/\text{mile}$
- GMR_n = 0.01113 ft

Distance between cable and neutral = $D_{nm} = 3$ in.

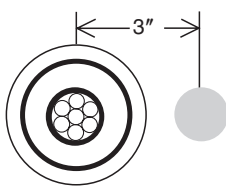
The resistance of the tape shield is computed according to Eq. (21.41):

$$r_{\text{shield}} = \frac{18.826}{d_s T} = \frac{18.826}{1.084 \times 8} = 2.1705 \Omega/\text{mile}$$

The GMR of the tape shield is computed according to Eq. (21.42):

$$\text{GMR}_{\text{shield}} = \frac{\frac{d_s}{2} - \frac{T}{2000}}{12} = \frac{\frac{1.084}{2} - \frac{8}{2000}}{12} = 0.0455 \text{ ft}$$

Using the relations defined in Eqs. (21.43) through (21.45) and Carson's equations results in a 3×3 primitive impedance matrix:



$$z_{\text{primitive}} = \begin{bmatrix} 1.0653 + j1.5088 & 0.0953 + j1.3377 & 0.0953 + j1.1309 \\ 0.0953 + j1.3377 & 2.2658 + j1.3377 & 0.0953 + j1.1309 \\ 0.0953 + j1.1309 & 0.0953 + j1.1309 & 0.7023 + j1.5085 \end{bmatrix} \Omega/\text{mile}$$

Applying Kron's reduction method will result in a single impedance that represents the equivalent single-phase impedance of the tape shield cable and the neutral conductor.

$$z_l = 1.3368 + j0.6028 \Omega/\text{mile}$$

FIGURE 21.10 Single-phase tape shield with neutral.

21.1.2 Shunt Admittance

When a high-voltage transmission line is less than 50 miles in length, the shunt capacitance of the line is typically ignored. For lightly loaded distribution lines, particularly underground lines, the shunt capacitance should be modeled.

The basic equation for the relationship between the charge on a conductor to the voltage drop between the conductor and ground is given by

$$Q_n = C_{ng} V_{ng} \quad (21.46)$$

where Q_n = charge on the conductor

C_{ng} = capacitance between the conductor and ground

V_{ng} = voltage between the conductor and ground

For a line consisting of n_{cond} (number of phase plus number of neutral) conductors, Eq. (21.46) can be written in condensed matrix form as

$$[Q] = [C][V] \quad (21.47)$$

where $[Q]$ = column vector of order n_{cond}

$[C]$ = $n_{cond} \times n_{cond}$ matrix

$[V]$ = column vector of order n_{cond}

Equation (21.47) can be solved for the voltages

$$[V] = [C]^{-1}[Q] = [P][Q] \quad (21.48)$$

$$\text{where } [P] = [C]^{-1} \quad (21.49)$$

21.1.2.1 Overhead Lines

The determination of the shunt admittance of overhead lines starts with the calculation of the “potential coefficient matrix” (Glover and Sarma, 1994). The elements of the matrix are determined by

$$P_{ii} = 11.17689 \times \ln \frac{S_{ii}}{RD_i} \quad (21.50)$$

$$P_{ij} = 11.17689 \times \ln \frac{S_{ij}}{D_{ij}} \quad (21.51)$$

See Fig. 21.2 for the following definitions.

S_{ii} = distance between a conductor and its image below ground in feet

S_{ij} = distance between conductor i and the image of conductor j below ground in feet

D_{ij} = overhead spacing between two conductors in feet

RD_i = radius of conductor i in feet

The potential coefficient matrix will be an $n_{cond} \times n_{cond}$ matrix. If one or more of the conductors is a grounded neutral, then the matrix must be reduced using the Kron method to an $n_{phase} \times n_{phase}$ matrix $[P_{abc}]$.

The inverse of the potential coefficient matrix will give the $n_{phase} \times n_{phase}$ capacitance matrix $[C_{abc}]$. The shunt admittance matrix is given by

$$[Y_{abc}] = j\omega[C_{abc}] \text{ } \mu\text{S/mile} \quad (21.52)$$

where $\omega = 2\pi f = 376.9911$

Example 21.4

Determine the shunt admittance matrix for the overhead line of [Example 21.1](#). Assume that the neutral conductor is 25 ft above ground.

Solution

For this configuration, the image spacing matrix is computed to be

$$[S] = \begin{bmatrix} 58 & 58.0539 & 58.4209 & 54.1479 \\ 58.0539 & 58 & 58.1743 & 54.0208 \\ 58.4209 & 58.1743 & 58 & 54.0833 \\ 54.1479 & 54.0208 & 54.0833 & 50 \end{bmatrix} \text{ ft}$$

The primitive potential coefficient matrix is computed to be

$$[P_{\text{primitive}}] = \begin{bmatrix} 84.56 & 35.1522 & 23.7147 & 25.2469 \\ 35.4522 & 84.56 & 28.6058 & 28.359 \\ 23.7147 & 28.6058 & 84.56 & 26.6131 \\ 25.2469 & 28.359 & 26.6131 & 85.6659 \end{bmatrix}$$

Kron reduce to a 3×3 matrix

$$[P] = \begin{bmatrix} 77.1194 & 26.7944 & 15.8714 \\ 26.7944 & 75.172 & 19.7957 \\ 15.8714 & 19.7957 & 76.2923 \end{bmatrix}$$

Invert $[P]$ to determine the shunt capacitance matrix

$$[Y_{\text{abc}}] = j376.9911[C_{\text{abc}}] = \begin{bmatrix} j5.6711 & -j1.8362 & -j0.7033 \\ -j1.8362 & j5.9774 & -j1.169 \\ -j0.7033 & -j1.169 & j5.391 \end{bmatrix} \mu\text{S/mile}$$

Multiply $[C_{\text{abc}}]$ by the radian frequency to determine the final three-phase shunt admittance matrix.

21.1.2.2 Underground Lines

Because the electric fields of underground cables are confined to the space between the phase conductor and its concentric neutral to tape shield, the calculation of the shunt admittance matrix requires only the determination of the “self” admittance terms.

21.1.2.3 Concentric Neutral

The self-admittance in $\mu\text{S/mile}$ for a concentric neutral cable is given by

$$Y_{cn} = j \frac{77.582}{\ln\left(\frac{R_b}{R_a}\right) - \frac{1}{k} \ln\left(\frac{kR_n}{R_b}\right)} \quad (21.53)$$

where R_b = radius of a circle to center of concentric neutral strands (ft)

R_a = radius of phase conductor (ft)

R_n = radius of concentric neutral strand (ft)

k = number of concentric neutral strands

Example 21.5

Determine the three-phase shunt admittance matrix for the concentric neutral line of [Example 21.2](#).

Solution

$$R_b = R = 0.0511 \text{ ft}$$

Diameter of the 250,000 AA phase conductor = 0.567 in.

$$R_a = \frac{0.567}{24} = 0.0236 \text{ ft}$$

Diameter of the #14 CU concentric neutral strand = 0.0641 in.

$$R_n = \frac{0.0641}{24} = 0.0027 \text{ ft}$$

Substitute into [Eq. \(21.53\)](#):

$$Y_{cn} = j \frac{77.582}{\ln\left(\frac{R_b}{R_a}\right) - \frac{1}{k} \ln\left(\frac{kR_n}{R_b}\right)} = j \frac{77.582}{\ln\left(\frac{0.0511}{0.0236}\right) - \frac{1}{13} \ln\left(\frac{13 \times 0.0027}{0.0511}\right)} = j96.8847$$

The three-phase shunt admittance matrix is:

$$[Y_{abc}] = \begin{bmatrix} j96.8847 & 0 & 0 \\ 0 & j96.8847 & 0 \\ 0 & 0 & j96.8847 \end{bmatrix} \mu\text{S/mile}$$

21.1.2.4 Tape Shield Cable

The shunt admittance in $\mu\text{S/mile}$ for tape shielded cables is given by

$$Y_{ts} = j \frac{77.586}{\ln\left(\frac{R_b}{R_a}\right)} \mu\text{S/mile} \quad (21.54)$$

where R_b = inside radius of the tape shield

R_a = radius of phase conductor

Example 21.6

Determine the shunt admittance of the single-phase tape shielded cable of [Example 21.3](#) in [Section 21.1.1](#).

Solution

$$R_b = \frac{d_s}{24} = \frac{1.084}{24} = 0.0452$$

The diameter of the 1/0 AA phase conductor = 0.368 in.

$$R_a = \frac{d_p}{24} = \frac{0.368}{24} = 0.0153$$

Substitute into Eq. (21.54):

$$Y_{ts} = j \frac{77.586}{\ln\left(\frac{R_b}{R_a}\right)} = j \frac{77.586}{\ln\left(\frac{0.0452}{0.0153}\right)} = j71.8169 \text{ } \mu\text{S/mile}$$

21.1.3 Line Segment Models

21.1.3.1 Exact Line Segment Model

The exact model of a three-phase line segment is shown in Fig. 21.11. For the line segment in Fig. 21.11, the equations relating the input (node n) voltages and currents to the output (node m) voltages and currents are

$$[VLG_{abc}]_n = [a][VLG_{abc}]_m + [b][I_{abc}]_m \quad (21.55)$$

$$[I_{abc}]_n = [c][VLG_{abc}]_m + [d][I_{abc}]_m \quad (21.56)$$

where

$$[a] = [U] - \frac{1}{2}[Z_{abc}][Y_{abc}] \quad (21.57)$$

$$[b] = [Z_{abc}] \quad (21.58)$$

$$[c] = [Y_{abc}] - \frac{1}{4}[Z_{abc}][Y_{abc}]^2 \quad (21.59)$$

$$[d] = [U] - \frac{1}{2}[Z_{abc}][Y_{abc}] \quad (21.60)$$

In Eqs. (21.57) through (21.60), the impedance matrix $[Z_{abc}]$ and the admittance matrix $[Y_{abc}]$ are defined earlier in this document.

Sometimes it is necessary to determine the voltages at node m as a function of the voltages at node n and the output currents at node m . The necessary equation is

$$[VLG_{abc}]_m = [A][VLG_{abc}]_n - [B][I_{abc}]_m \quad (21.61)$$

where

$$[A] = \left([U] + \frac{1}{2}[Z_{abc}][Y_{abc}] \right)^{-1} \quad (21.62)$$

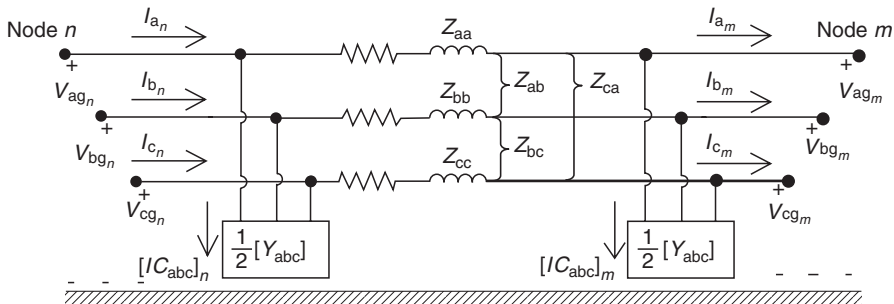


FIGURE 21.11 Three-phase line segment model.

$$[B] = \left([U] + \frac{1}{2} [Z_{abc}] [Y_{abc}] \right)^{-1} [Z_{abc}] \quad (21.63)$$

$$[U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (21.64)$$

In many cases the shunt admittance is so small that it can be neglected. However, for all underground cables and for overhead lines longer than 15 miles, it is recommended that the shunt admittance be included. When the shunt admittance is neglected, the [a], [b], [c], [d], [A], and [B] matrices become

$$[a] = [U] \quad (21.65)$$

$$[b] = [Z_{abc}] \quad (21.66)$$

$$[c] = [0] \quad (21.67)$$

$$[d] = [U] \quad (21.68)$$

$$[A] = [U] \quad (21.69)$$

$$[B] = [Z_{abc}] \quad (21.70)$$

When the shunt admittance is neglected, Eqs. (21.55), (21.56), and (21.61) become

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}] [I_{abc}]_m \quad (21.71)$$

$$[I_{abc}]_n = [I_{abc}]_m \quad (21.72)$$

$$[VLG_{abc}]_m = [VLG_{abc}]_n - [Z_{abc}] [I_{abc}]_m \quad (21.73)$$

If an accurate determination of the voltage drops down a line segment is to be made, it is essential that the phase impedance matrix $[Z_{abc}]$ be computed based upon the actual configuration and phasing of the overhead or underground lines. No assumptions should be made, such as transposition. The reason for this is best demonstrated by an example.

Example 21.7

The phase impedance matrix for the line configuration in [Example 21.1](#) was computed to be

$$[z_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

Assume that a 12.47 kV substation serves a load 1.5 miles from the substation. The metered output at the substation is balanced 10,000 kVA at 12.47 kV and 0.9 lagging power factor. Compute the three-phase line-to-ground voltages at the load end of the line and the voltage unbalance at the load.

Solution

The line-to-ground voltages and line currents at the substation are

$$[VLG_{abc}] = \begin{bmatrix} 7200 \angle 0 \\ 7200 \angle -120 \\ 7200 \angle 120 \end{bmatrix} \quad [I_{abc}]_n = \begin{bmatrix} 463 \angle -25.84 \\ 463 \angle -145.84 \\ 463 \angle 94.16 \end{bmatrix}$$

Solve Eq. (21.71) for the load voltages:

$$[VLG_{abc}]_m = [VLG_{abc}]_n - 1.5[Z_{abc}][I_{abc}]_n = \begin{bmatrix} 6761.10 \angle 2.32 \\ 6877.7 \angle -122.43 \\ 6836.33 \angle 117.21 \end{bmatrix}$$

The voltage unbalance at the load using the NEMA definition is

$$V_{\text{unbalance}} = \frac{\max(V_{\text{deviation}})}{V_{\text{avg}}} 100 = 0.937\%$$

The point of Example 21.7 is to demonstrate that even though the system is perfectly balanced at the substation, the unequal mutual coupling between the phases results in a significant voltage unbalance at the load; significant because NEMA requires that induction motors be derated when the voltage unbalance is 1% or greater.

21.1.3.2 Approximate Line Segment Model

Many times the only data available for a line segment will be the positive and zero sequence impedances. An approximate three-phase line segment model can be developed by applying the “reverse impedance transformation” from symmetrical component theory.

Using the known positive and zero sequence impedances, the “sequence impedance matrix” is given by

$$[Z_{\text{seq}}] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_+ & 0 \\ 0 & 0 & Z_+ \end{bmatrix} \quad (21.74)$$

The reverse impedance transformation results in the following “approximate phase impedance matrix:”

$$[Z_{\text{approx}}] = [A_s][Z_{\text{seq}}][A_s]^{-1} = \frac{1}{3} \begin{bmatrix} (2Z_+ - Z_0) & (Z_0 - Z_+) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (2Z_+ - Z_0) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (Z_0 - Z_+) & (2Z_+ - Z_0) \end{bmatrix} \quad (21.75)$$

Notice that the approximate phase impedance matrix is characterized by the three diagonal terms being equal and all mutual terms being equal. This is the same result that is achieved if the line is assumed to be transposed. Substituting the approximate phase impedance matrix into Eq. (21.71) results in

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}_n = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}_m + \frac{1}{3} \begin{bmatrix} (2Z_+ - Z_0) & (Z_0 - Z_+) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (2Z_+ - Z_0) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (Z_0 - Z_+) & (2Z_+ - Z_0) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_n \quad (21.76)$$

Equation (21.76) can be expanded and an equivalent circuit for the approximate line segment model can be developed. This approximate model is shown in Fig. 21.12.

The errors made by using this approximate line segment model are demonstrated in the following example.

Example 21.8

For the line of Example 21.7, the positive and zero sequence impedances were determined to be

$$\begin{aligned} Z_+ &= 0.3061 + j0.6270 \, \Omega/\text{mile} \\ Z_0 &= 0.7735 + j1.9373 \, \Omega/\text{mile} \end{aligned}$$

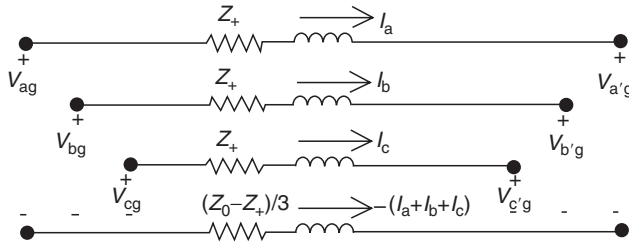


FIGURE 21.12 Approximate line segment model.

Solution

The sequence impedance matrix is

$$[Z_{\text{seq}}] = \begin{bmatrix} 0.7735 + j1.9373 & 0 & 0 \\ 0 & 0.3061 + j0.6270 & 0 \\ 0 & 0 & 0.3061 + j0.6270 \end{bmatrix}$$

Performing the reverse impedance transformation results in the approximate phase impedance matrix.

$$[Z_{\text{approx}}] = [A_s][Z_{\text{seq}}][A_s]^{-1} = \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix}$$

Note in the approximate phase impedance matrix that the three diagonal terms are equal and all of the mutual terms are equal.

Use the approximate impedance matrix to compute the load voltage and voltage unbalance as specified in [Example 21.1](#).

Note that the voltages are computed to be balanced. In the previous example it was shown that when the line is modeled accurately, there is a voltage unbalance of almost 1%.

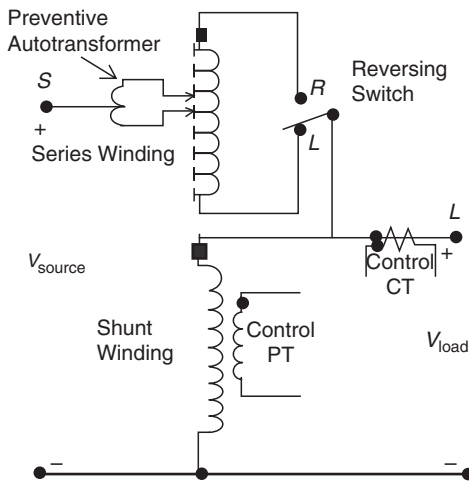


FIGURE 21.13 Type B step-voltage regulator.

21.1.4 Step-Voltage Regulators

A step-voltage regulator consists of an autotransformer and a load tap changing mechanism. The voltage change is obtained by changing the taps of the series winding of the autotransformer. The position of the tap is determined by a control circuit (line drop compensator). Standard step regulators contain a reversing switch enabling a $\pm 10\%$ regulator range, usually in 32 steps. This amounts to a 5/8% change per step or 0.75 V change per step on a 120 V base.

A type B step-voltage regulator is shown in Fig. 21.13. There is also a type A step-voltage regulator where the load and source sides of the regulator are reversed from that shown in Fig. 21.13.

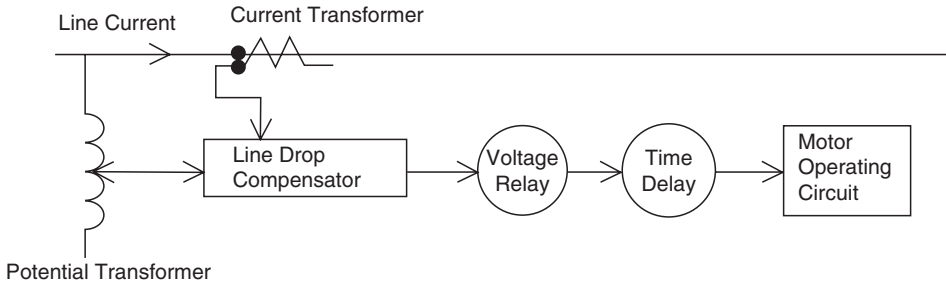


FIGURE 21.14 Regulator control circuit.

Since the type B regulator is more common, the remainder of this section will address the type B step-voltage regulator.

The tap changing is controlled by a control circuit shown in the block diagram of Fig. 21.14. The control circuit requires the following settings:

1. **Voltage Level**—The desired voltage (on 120 V base) to be held at the “load center.” The load center may be the output terminal of the regulator or a remote node on the feeder.
2. **Bandwidth**—The allowed variance of the load center voltage from the set voltage level. The voltage held at the load center will be $\pm \frac{1}{2}$ of the bandwidth. For example, if the voltage level is set to 122 V and the bandwidth set to 2 V, the regulator will change taps until the load center voltage lies between 121 and 123 V.
3. **Time Delay**—Length of time that a raise or lower operation is called for before the actual execution of the command. This prevents taps changing during a transient or short time change in current.
4. **Line Drop Compensator**—Set to compensate for the voltage drop (line drop) between the regulator and the load center. The settings consist of R and X settings in volts corresponding to the equivalent impedance between the regulator and the load center. This setting may be zero if the regulator output terminals are the load center.

The rating of a regulator is based on the kVA transformed, not the kVA rating of the line. In general this will be 10% of the line rating since rated current flows through the series winding that represents the $\pm 10\%$ voltage change.

21.1.4.1 Voltage Regulator in the Raise Position

Figure 21.15 shows a detailed and abbreviated drawing of a type B regulator in the raise position. The defining voltage and current equations for the type B regulator in the raise position are as follows:

$$\begin{array}{ll} \text{Voltage equations} & \text{Current equations} \\ \frac{V_1}{N_1} = \frac{V_2}{N_2} & N_1 I_1 = N_2 I_2 \end{array} \quad (21.77)$$

$$V_S = V_1 - V_2 \quad I_L = I_S - I_1 \quad (21.78)$$

$$V_L = V_1 \quad I_2 = I_S \quad (21.79)$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} V_L \quad I_1 = \frac{N_2}{N_1} I_2 = \frac{N_2}{N_1} I_S \quad (21.80)$$

The defining voltage and current equations for a regulator in the lower position are as follows:

Voltage equations

Current equations

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad N_1 I_1 = N_2 I_2 \quad (21.84)$$

$$V_S = V_1 + V_2 \quad I_L = I_S - I_1 \quad (21.85)$$

$$V_L = V_1 \quad I_2 = -I_S \quad (21.86)$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} V_L \quad I_1 = \frac{N_2}{N_1} I_2 = \frac{N_2}{N_1} (-I_S) \quad (21.87)$$

$$V_S = \left(1 + \frac{N_2}{N_1}\right) V_1 \quad I_L = \left(1 + \frac{N_2}{N_1}\right) I_S \quad (21.88)$$

$$V_S = a_R V_L \quad I_L = a_R I_S \quad (21.89)$$

$$a_R = 1 + \frac{N_2}{N_1} \quad (21.90)$$

Equations (21.83) and (21.90) give the value of the effective regulator ratio as a function of the ratio of the number of turns on the series winding (N_2) to the number of turns on the shunt winding (N_1). The actual turns ratio of the windings is not known. However, the particular position will be known. Equations (21.83) and (21.90) can be modified to give the effective regulator ratio as a function of the tap position. Each tap changes the voltage by 5/8% or 0.00625 per unit. On a 120 V base, each step change results in a change of voltage of 0.75 V. The effective regulator ratio can be given by

$$a_R = 1 \mp 0.00625 \cdot \text{Tap} \quad (21.91)$$

In Eq. (21.91), the minus sign applies to the “raise” position and the positive sign for the “lower” position.

21.1.4.3 Line Drop Compensator

The changing of taps on a regulator is controlled by the “line drop compensator.” Figure 21.17 shows a simplified sketch of the compensator circuit and how it is connected to the circuit through a potential transformer and a current transformer.

The purpose of the line drop compensator is to model the voltage drop of the distribution line from the regulator to the load center. Typically, the compensator circuit is modeled on a 120 V base. This requires the potential transformer to transform rated voltage (line-to-neutral or line-to-line) down to 120 V. The current transformer turns ratio ($CT_p:CT_s$) where the primary rating (CT_p) will typically be the rated current of the feeder. The setting that is most critical is that of R' and X' . These values must represent the equivalent impedance from the regulator to the load center. Knowing the equivalent impedance in Ohms from the regulator to the load center ($R_{\text{line_ohms}}$ and $X_{\text{line_ohms}}$), the required value for the compensator settings are calibrated in volts and determined by

$$R'_{\text{volts}} + jX'_{\text{volts}} = (R_{\text{line_ohms}} + jX_{\text{line_ohms}}) \cdot \frac{CT_p}{N_{\text{pt}}} \text{ V} \quad (21.92)$$

The value of the compensator settings in ohms is determined by

$$R'_{\text{ohms}} + jX'_{\text{ohms}} = \frac{R'_{\text{volts}} + jX'_{\text{volts}}}{CT_s} \Omega \quad (21.93)$$

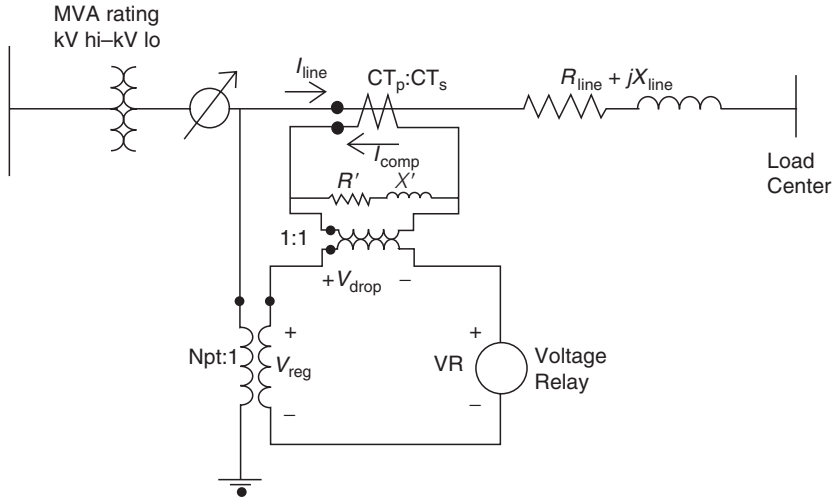


FIGURE 21.17 Line drop compensator circuit.

It is important to understand that the value of $R_{\text{line_ohms}} + jX_{\text{line_ohms}}$ is not the impedance of the line between the regulator and the load center. Typically the load center is located down the primary main feeder after several laterals have been tapped. As a result, the current measured by the CT of the regulator is not the current that flows all the way from the regulator to the load center. The proper way to determine the line impedance values is to run a power-flow program of the feeder without the regulator operating. From the output of the program, the voltages at the regulator output and the load center are known. Now the “equivalent” line impedance can be computed as

$$R_{\text{line}} + jX_{\text{line}} = \frac{V_{\text{regulator_output}} - V_{\text{load_center}}}{I_{\text{line}}} \Omega \quad (21.94)$$

In Eq. (21.94), the voltages must be specified in system volts and the current in system amps.

21.1.4.4 Wye Connected Regulators

Three single-phase regulators connected in wye are shown in Fig. 21.18. In Fig. 21.18 the polarities of the windings are shown in the raise position. When the regulator is in the lower position, a reversing switch will have reconnected the series winding so that the polarity on the series winding is now at the output terminal.

Regardless of whether the regulator is raising or lowering the voltage, the following equations apply:

21.1.4.5 Voltage Equations

$$\begin{bmatrix} V_{An} \\ V_{Bn} \\ V_{Cn} \end{bmatrix} = \begin{bmatrix} a_{R-a} & 0 & 0 \\ 0 & a_{R-b} & 0 \\ 0 & 0 & a_{R-c} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \quad (21.95)$$

Equation (21.95) can be written in condensed form as

$$[VLN_{ABC}] = [aRV_{abc}][VLN_{abc}] \quad (21.96)$$

also

$$[VLN_{abc}] = [aRV_{ABC}][VLN_{ABC}] \quad (21.97)$$

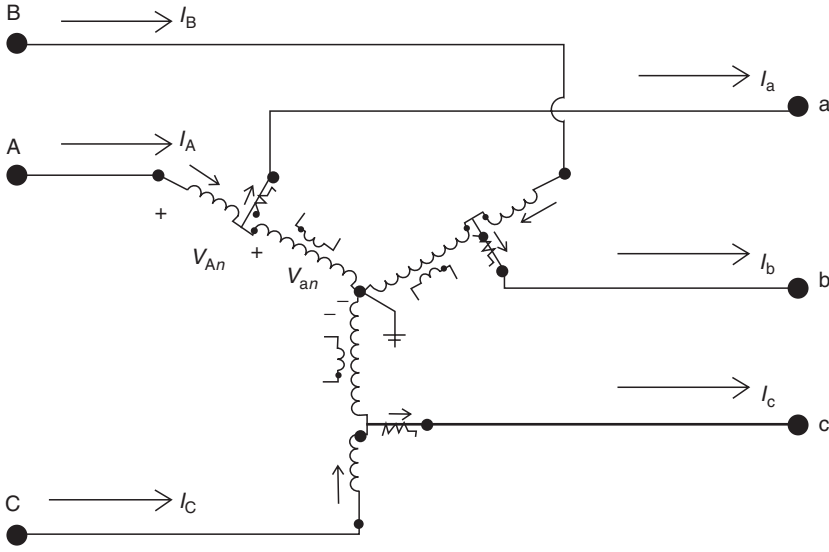


FIGURE 21.18 Wye connected type B regulators.

where

$$[aRV_{ABC}] = [aRV_{abc}]^{-1} \quad (21.98)$$

21.1.4.6 Current Equations

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R-a}} & 0 & 0 \\ 0 & \frac{1}{a_{R-b}} & 0 \\ 0 & 0 & \frac{1}{a_{R-c}} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (21.99)$$

or

$$[I_{ABC}] = [aRI_{abc}] [I_{abc}] \quad (21.100)$$

also

$$[I_{abc}] = [aRI_{ABC}] [I_{ABC}] \quad (21.101)$$

where

$$[aRI_{ABC}] = [aRI_{abc}]^{-1} \quad (21.102)$$

where $0.9 \leq a_{R-abc} \leq 1.1$ in 32 steps of 0.625% per step (0.75 V/step on 120 V base).

Note: The effective turn ratios (a_{R-a} , a_{R-b} , and a_{R-c}) can take on different values when three single-phase regulators are connected in wye. It is also possible to have a three-phase regulator connected in wye where the voltage and current are sampled on only one phase and then all three phases are changed by the same value of aR (number of taps).

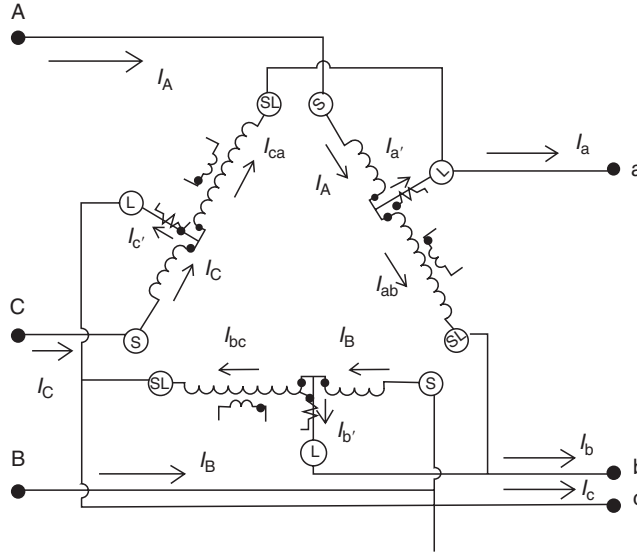


FIGURE 21.19 Delta connected type B regulators.

21.1.4.7 Closed Delta Connected Regulators

Three single-phase regulators can be connected in a closed delta as shown in Fig. 21.19. In the figure, the regulators are shown in the raise position. The closed delta connection is typically used in three-wire delta feeders. Note that the potential transformers for this connection are monitoring the load side line-to-line voltages and the current transformers are monitoring the load side line currents.

Applying the basic voltage and current Eqs. (21.77) through (21.83) of the regulator in the raise position, the following voltage and current relations are derived for the closed delta connection.

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \begin{bmatrix} a_{R_ab} & 1 - a_{R_bc} & 0 \\ 0 & a_{R_bc} & 1 - a_{R_ca} \\ 1 - a_{R_ab} & 0 & a_{R_ca} \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \quad (21.103)$$

Equation (21.101) in abbreviated form can be written as

$$[VLL_{ABC}] = [aRVD_{abc}][VLL_{abc}] \quad (21.104)$$

When the load side voltages are known, the source side voltages can be determined by

$$[VLL_{abc}] = [aRVD_{ABC}][VLL_{ABC}] \quad (21.105)$$

where

$$[aRVD_{ABC}] = [aRVD_{abc}]^{-1} \quad (21.106)$$

In a similar manner, the relationships between the load side and source side line currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} a_{R_ab} & 0 & 1 - a_{R_ca} \\ 1 - a_{R_ab} & a_{R_bc} & 0 \\ 0 & 1 - a_{R_bc} & a_{R_ca} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (21.107)$$

or

$$[I_{abc}] = [AID_{ABC}][I_{ABC}] \quad (21.108)$$

also

$$[I_{ABC}] = [AID_{abc}][I_{abc}] \quad (21.109)$$

where

$$[IAD_{abc}] = [IAD_{ABC}]^{-1} \quad (21.110)$$

The closed delta connection can be difficult to apply. Note in both the voltage and current equations that a change of the tap position in one regulator will affect voltages and currents in two phases. As a result, increasing the tap in one regulator will affect the tap position of the second regulator. In most cases the bandwidth setting for the closed delta connection will have to be wider than that for wye connected regulators.

21.1.4.8 Open Delta Connection

Two single-phase regulators can be connected in the “open” delta connection. Shown in Fig. 21.20 is an open delta connection where two single-phase regulators have been connected between phases AB and CB.

Two other open connections can also be made where the single-phase regulators are connected between phases BC and AC and also between phases CA and BA.

The open delta connection is typically applied to three-wire delta feeders. Note that the potential transformers monitor the line-to-line voltages and the current transformers monitor the line currents. Once again, the basic voltage and current relations of the individual regulators are used to determine the relationships between the source side and load side voltages and currents.

For all three open connections, the following general equations will apply:

$$[VLL_{ABC}] = [aRV_{abc}][VLL_{abc}] \quad (21.111)$$

$$[VLL_{abc}] = [aRV_{ABC}][VLL_{ABC}] \quad (21.112)$$

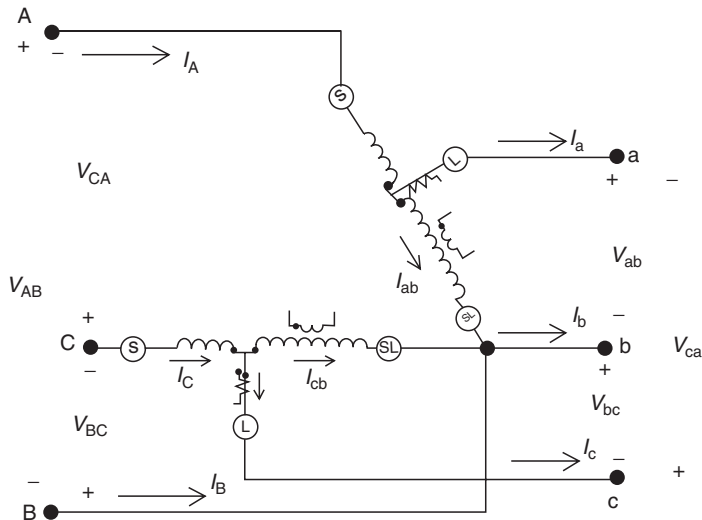


FIGURE 21.20 Open delta type B regulator connection.

$$[I_{ABC}] = [aRI_{abc}][I_{abc}] \quad (21.113)$$

$$[I_{abc}] = [aRI_{ABC}][I_{ABC}] \quad (21.114)$$

The matrices for the three open connections are defined as follows:
Phases AB and CB

$$[aRV_{abc}] = \begin{bmatrix} a_{R_A} & 0 & 0 \\ 0 & a_{R_C} & 0 \\ -a_{R_A} & -a_{R_C} & 0 \end{bmatrix} \quad (21.115)$$

$$[aRV_{ABC}] = \begin{bmatrix} \frac{1}{a_{R_A}} & 0 & 0 \\ 0 & \frac{1}{a_{R_C}} & 0 \\ -\frac{1}{a_{R_A}} & -\frac{1}{a_{R_C}} & 0 \end{bmatrix} \quad (21.116)$$

$$[aRI_{abc}] = \begin{bmatrix} \frac{1}{a_{R_A}} & 0 & 0 \\ -\frac{1}{a_{R_A}} & 0 & -\frac{1}{a_{R_C}} \\ 0 & 0 & \frac{1}{a_{R_C}} \end{bmatrix} \quad (21.117)$$

$$[aRI_{ABC}] = \begin{bmatrix} a_{R_A} & 0 & 0 \\ -a_{R_A} & 0 & a_{R_C} \\ 0 & 0 & a_{R_C} \end{bmatrix} \quad (21.118)$$

Phases BC and AC

$$[aRV_{abc}] = \begin{bmatrix} 0 & -a_{R_B} & -a_{R_A} \\ 0 & a_{R_B} & 0 \\ 0 & 0 & a_{R_A} \end{bmatrix} \quad (21.119)$$

$$[aRV_{ABC}] = \begin{bmatrix} 0 & -\frac{1}{a_{R_B}} & -\frac{1}{a_{R_A}} \\ 0 & \frac{1}{a_{R_B}} & 0 \\ 0 & 0 & \frac{1}{a_{R_A}} \end{bmatrix} \quad (21.120)$$

$$[aRI_{abc}] = \begin{bmatrix} \frac{1}{a_{R_A}} & 0 & 0 \\ 0 & \frac{1}{a_{R_B}} & 0 \\ -\frac{1}{a_{R_A}} & -\frac{1}{a_{R_B}} & 0 \end{bmatrix} \quad (21.121)$$

$$[aRI_{ABC}] = \begin{bmatrix} a_{R_A} & 0 & 0 \\ 0 & a_{R_B} & 0 \\ -a_{R_A} & -a_{R_B} & 0 \end{bmatrix} \quad (21.122)$$

Phases CA and BA

$$[aRV_{abc}] = \begin{bmatrix} a_{R_B} & 0 & 0 \\ -a_{R_B} & 0 & -a_{R_C} \\ 0 & 0 & a_{R_C} \end{bmatrix} \quad (21.123)$$

$$[aRV_{ABC}] = \begin{bmatrix} \frac{1}{a_{R_B}} & 0 & 0 \\ -\frac{1}{a_{R_B}} & 0 & -\frac{1}{a_{R_C}} \\ 0 & 0 & \frac{1}{a_{R_C}} \end{bmatrix} \quad (21.124)$$

$$[aRI_{abc}] = \begin{bmatrix} 0 & -\frac{1}{a_{R_B}} & -\frac{1}{a_{R_C}} \\ 0 & \frac{1}{a_{R_B}} & 0 \\ 0 & 0 & \frac{1}{a_{R_C}} \end{bmatrix} \quad (21.125)$$

$$[aRI_{ABC}] = \begin{bmatrix} 0 & -a_{R_B} & -a_{R_C} \\ 0 & a_{R_B} & 0 \\ 0 & 0 & a_{R_C} \end{bmatrix} \quad (21.126)$$

21.1.4.9 Generalized Equations

The voltage regulator models used in power-flow studies are generalized for the various connections in a form similar to the ABCD parameters that are used in transmission line analysis. The general form of the power-flow models in matrix form is

$$[V_{ABC}] = [a][V_{abc}] + [b][I_{abc}] \quad (21.127)$$

$$[I_{ABC}] = [c][V_{abc}] + [d][I_{abc}] \quad (21.128)$$

$$[V_{abc}] = [A][V_{ABC}] - [B][I_{abc}] \quad (21.129)$$

Depending upon the connection, the matrices $[V_{ABC}]$ and $[V_{abc}]$ can be either line-to-line or line-to-ground. The current matrices represent the line currents regardless of the regulator connection. For all voltage regulator connections, the generalized constants are defined as

$$[a] = [aRV_{abc}] \quad (21.130)$$

$$[b] = [0] \quad (21.131)$$

$$[c] = [0] \quad (21.132)$$

$$[d] = [aRI_{abc}] \quad (21.133)$$

$$[A] = [aRV_{ABC}] \quad (21.134)$$

$$[B] = [0] \quad (21.135)$$

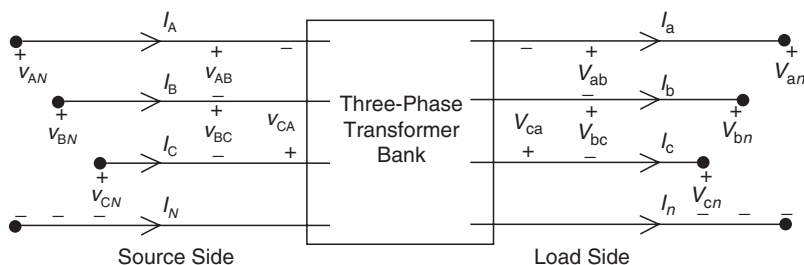


FIGURE 21.21 General transformer bank.

21.1.5 Transformer Bank Connections

Unique models of three-phase transformer banks applicable to radial distribution feeders have been developed (Kersting, 1999). Models for the following three-phase connections are included in this document:

- Delta-grounded wye
- Grounded wye-delta
- Ungrounded wye-delta
- Grounded wye-grounded wye
- Delta-delta

Figure 21.21 defines the various voltages and currents for the transformer bank models. The models can represent a step-down (source side to load side) or a step-up (source side to load side) transformer bank. The notation is such that the capital letters A,B,C,N will always refer to the source side of the bank and the lower case letters a,b,c,n will always refer to the load side of the bank. It is assumed that all variations of the wye-delta connections are connected in the “American Standard Thirty Degree” connection. The standard is such that:

Step-down connection

$$\begin{aligned} V_{AB} &\text{ leads } V_{ab} \text{ by } 30^\circ \\ I_A &\text{ leads } I_a \text{ by } 30^\circ \end{aligned}$$

Step-up connection

$$\begin{aligned} V_{ab} &\text{ leads } V_{AB} \text{ by } 30^\circ \\ I_a &\text{ leads } I_A \text{ by } 30^\circ \end{aligned}$$

21.1.5.1 Generalized Equations

The models to be used in power-flow studies are generalized for the connections in a form similar to the ABCD parameters that are used in transmission line analysis. The general form of the power-flow models in matrix form are

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}] \quad (21.136)$$

$$[I_{ABC}] = [c_t][V_{abc}] + [d_t][I_{abc}] \quad (21.137)$$

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}] \quad (21.138)$$

In Eqs. (21.136) through (21.138), the matrices $[VLN_{ABC}]$ and $[VLN_{abc}]$ will be the equivalent line-to-neutral voltages on delta and ungrounded wye connections and the line-to-ground voltages for grounded wye connections.

When the “ladder technique” or “sweep” iterative method is used, the “forward” sweep is assumed to be from the source working toward the remote nodes. The “backward” sweep will be working from the remote nodes toward the source node.

21.1.5.2 Common Variable and Matrices

All transformer models will use the following common variable and matrices:

- Transformer turns ratio

$$n_t = \frac{V_{\text{rated_source}}}{V_{\text{rated_load}}} \quad (21.139)$$

where $V_{\text{rated_source}}$ = transformer winding rating on the source side. Line-to-line voltage for delta connections and line-to-neutral for wye connections.

$V_{\text{rated_load}}$ = transformer winding rating on the load side. Line-to-line voltage for delta connections and line-to-neutral for wye connections.

Note that the transformer “winding” ratings may be either line-to-line or line-to-neutral, depending upon the connection. The winding ratings can be specified in actual volts or per-unit volts using the appropriate base line-to-neutral voltages.

- Source to load matrix voltage relations:

$$[V_{ABC}] = [AV][V_{abc}] \quad (21.140)$$

The voltage matrices may be line-to-line or line-to-neutral voltages depending upon the connection.

- Load to source matrix current relations:

$$[I_{abc}] = [AI][I_{ABC}] \quad (21.141)$$

The current matrices may be line currents or delta currents depending upon the connection.

- Transformer impedance matrix:

$$[Z_{t_{abc}}] = \begin{bmatrix} Z_{t_a} & 0 & 0 \\ 0 & Z_{t_b} & 0 \\ 0 & 0 & Z_{t_c} \end{bmatrix} \quad (21.142)$$

The impedance elements in the matrix will be the per-unit impedance of the transformer windings on the load side of the transformer whether it is connected in wye or delta.

- Symmetrical component transformation matrix:

$$[A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \quad (21.143)$$

where $a_s = 1 \angle 120$

- Phaseshift matrix

$$[T_s] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_s^* & 0 \\ 0 & 0 & t_s \end{bmatrix} \quad (21.144)$$

where $t_s = \frac{1}{\sqrt{3}} / 30$

- Matrix to convert line-to-line voltages to equivalent line-to-neutral voltages:

$$[W] = [A_s][T_s][A_s]^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad (21.145)$$

Example: $[VLN] = [W][VLL]$

- Matrix to convert delta currents into line currents:

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (21.146)$$

Example: $[I_{abc}] = [DI][ID_{abc}]$

- Matrix to convert line-to-ground or line-to-neutral voltages to line-to-line voltages:

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (21.147)$$

Example: $[VLL_{abc}] = [D][VLN_{abc}]$

21.1.5.3 Per-Unit System

All transformer models were developed so that they can be applied using either “actual” or “per-unit” values of voltages, currents, and impedances. When the per-unit system is used, all per-unit voltages (line-to-line and line-to-neutral) use the line-to-neutral base as the base voltage. In other words, for a balanced set of three-phase voltages, the per-unit line-to-neutral voltage magnitude will be 1.0 at rated voltage and the per-unit line-to-line voltage magnitude will be the $\sqrt{3}$. In a similar fashion, all currents (line currents and delta currents) are based on the base line current. Again, $\sqrt{3}$ relationship will exist between the line and delta currents under balanced conditions. The base line impedance will be used for all line impedances and for wye and delta connected transformer impedances. There will be different base values on the two sides of the transformer bank.

Base values are computed following the steps listed below:

- Select a base three-phase kVA_{base} and the rated line-to-line voltage, $kVLL_{source}$, on the source side as the base line-to-line voltage.
- Based upon the voltage ratings of the transformer bank, determine the rated line-to-line voltage, $kVLL_{load}$, on the load side.
- Determine the transformer ratio, a_x , as

$$a_x = \frac{kVLL_{source}}{kVLL_{load}} \quad (21.148)$$

- The source side base values are computed as

$$kVLN_s = \frac{kVLL_s}{\sqrt{3}} \quad (21.149)$$

$$I_S = \frac{kVA_{\text{base}}}{\sqrt{3}kVLL_{\text{source}}} \quad (21.150)$$

$$Z_S = \frac{kVLL_{\text{source}}^2 1000}{kVA_B} \quad (21.151)$$

- The load side base values are computed by

$$kVNL_L = \frac{kVNL_S}{a_x} \quad (21.152)$$

$$I_L = a_x I_S \quad (21.153)$$

$$Z_L = \frac{Z_S}{a_x^2} \quad (21.154)$$

The matrices $[a_t]$, $[b_t]$, $[c_t]$, $[d_t]$, $[A_t]$, and $[B_t]$ [see Eqs. (21.136) through (21.138)] for each connection are defined as follows:

21.1.5.4 Matrix Definitions

21.1.5.4.1 Delta–Grounded Wye

Backward sweep:

$$\begin{aligned} [VLN_{ABC}] &= [a_t][VLG_{abc}] + [b_t][I_{abc}] \\ [I_{ABC}] &= [c_t][VLG_{abc}] + [d_t][I_{abc}] \end{aligned}$$

Forward sweep:

$$[VLG_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}]$$

The matrices used for the step-down connection are

$$\begin{aligned} [a_t] &= \frac{-n_t}{3} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \\ [b_t] &= \frac{-n_t}{3} \begin{bmatrix} 0 & 2Z_{t_b} & Z_{t_c} \\ Z_{t_a} & 0 & 2Z_{t_c} \\ 2Z_{t_a} & Z_{t_b} & 0 \end{bmatrix} \\ [c_t] &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ [d_t] &= \frac{1}{n_t} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$[A_t] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[B_t] = \begin{bmatrix} Z_{t_a} & 0 & 0 \\ 0 & Z_{t_b} & 0 \\ 0 & 0 & Z_{t_c} \end{bmatrix}$$

21.1.5.4.2 Ungrounded Wye–Delta

Power-flow equations:

Backward sweep:

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}]$$

$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}]$$

Forward sweep:

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}]$$

Matrices used for the step-down connection are

$$[a_t] = n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[b_t] = \frac{n_t}{3} \begin{bmatrix} Z_{t_{ab}} & -Z_{t_{ab}} & 0 \\ Z_{t_{bc}} & 2Z_{t_{bc}} & 0 \\ -2Z_{t_{ca}} & -Z_{t_{ca}} & 0 \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{3n_t} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix}$$

$$[A_t] = \frac{1}{3n_t} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$[B_t] = \frac{1}{9} \begin{bmatrix} 2Z_{t_{ab}} + Z_{t_{bc}} & 2Z_{t_{bc}} - 2Z_{t_{ab}} & 0 \\ 2Z_{t_{bc}} - 2Z_{t_{ca}} & 4Z_{t_{bc}} - Z_{t_{ca}} & 0 \\ Z_{t_{ab}} - 4Z_{t_{ca}} & -Z_{t_{ab}} + 2Z_{t_{ca}} & 0 \end{bmatrix}$$

where $Z_{t_{ab}}$, $Z_{t_{bc}}$, and $Z_{t_{ca}}$ are the transformer impedances inside the delta secondary connection.

21.1.5.4.3 Grounded Wye–Delta

Power-flow equations:

Backward sweep:

$$\begin{aligned} [VLG_{ABC}] &= [a_t][VLN_{abc}] + [b_t][I_{abc}] \\ [I_{ABC}] &= [c_t][VLN_{abc}] + [d_t][I_{abc}] \end{aligned}$$

Forward sweep:

$$[VLN_{abc}] = [A_t][VLG_{ABC}] - [B_t][I_{abc}]$$

The matrices used for the step-down connection are

$$\begin{aligned} [a_t] &= n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \\ [b_t] &= \frac{n_t}{Z_{tab} + Z_{tbc} + Z_{tca}} \begin{bmatrix} Z_{tab}Z_{tca} & -Z_{tab}Z_{tbc} & 0 \\ Z_{tbc}Z_{tca} & Z_{tbc}(Z_{tca} + Z_{tab}) & 0 \\ Z_{tca}(-Z_{tab} - Z_{tbc}) & -Z_{tbc}Z_{tca} & 0 \end{bmatrix} \\ [c_t] &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ [d_t] &= \frac{1}{n_t(Z_{tab} + Z_{tbc} + Z_{tca})} \begin{bmatrix} Z_{tca} & -Z_{tbc} & 0 \\ Z_{tca} & Z_{tab} + Z_{tca} & 0 \\ -Z_{tab} - Z_{tbc} & -Z_{tca} & 0 \end{bmatrix} \\ [A_t] &= \frac{1}{3n_t} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \\ [B_t] &= \frac{1}{3 \sum Z_t} \begin{bmatrix} 2Z_{tab}Z_{tca} + Z_{tbc}Z_{tca} & -2Z_{tab}Z_{tbc} + Z_{tbc}(Z_{tab} + Z_{tca}) & 0 \\ 2Z_{tbc}Z_{tca} - Z_{tbc}(Z_{tab} + Z_{tbc}) & 2Z_{tbc}(Z_{tab} + Z_{tca}) - Z_{tbc}Z_{tca} & 0 \\ Z_{tab}Z_{tca} - 2Z_{tca}(Z_{tab} + Z_{tbc}) & -Z_{tab}Z_{tbc} - 2Z_{tbc}Z_{tca} & 0 \end{bmatrix} \end{aligned}$$

where

$$\sum Z_t = Z_{tab} + Z_{tbc} + Z_{tca}$$

21.1.5.4.4 The Grounded Wye–Grounded Wye Connection

Power-flow equations:

Backward sweep:

$$\begin{aligned} [VLG_{ABC}] &= [a_t][VLG_{abc}] + [b_t][I_{abc}] \\ [I_{ABC}] &= [c_t][VLG_{abc}] + [d_t][I_{abc}] \end{aligned}$$

Forward sweep:

$$[VLG_{abc}] = [A_t][VLG_{ABC}] - [B_t][I_{abc}]$$

The matrices used are

$$[a_t] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[b_t] = n_t \begin{bmatrix} Z_{t_a} & 0 & 0 \\ 0 & Z_{t_b} & 0 \\ 0 & 0 & Z_{t_c} \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A_t] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B_t] = \begin{bmatrix} Z_{t_a} & 0 & 0 \\ 0 & Z_{t_b} & 0 \\ 0 & 0 & Z_{t_c} \end{bmatrix}$$

21.1.5.4.5 Delta–Delta

Power-flow equations:

Backward sweep:

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}]$$

$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}]$$

Forward sweep:

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}]$$

The matrices used are

$$[a_t] = \frac{n_t}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$[b_t] = \frac{n_t}{3 \sum Z_t} \begin{bmatrix} 2Z_{t_{ab}}Z_{t_{ca}} + Z_{t_{bc}}Z_{t_{ca}} & -2Z_{t_{ab}}Z_{t_{bc}} + Z_{t_{bc}}(Z_{t_{ab}} + Z_{t_{ca}}) & 0 \\ 2Z_{t_{bc}}Z_{t_{ca}} - Z_{t_{bc}}(Z_{t_{ab}} + Z_{t_{bc}}) & 2Z_{t_{bc}}(Z_{t_{ab}} + Z_{t_{ca}}) - Z_{t_{bc}}Z_{t_{ca}} & 0 \\ Z_{t_{ab}}Z_{t_{ca}} - 2Z_{t_{ca}}(Z_{t_{ab}} + Z_{t_{bc}}) & -Z_{t_{ab}}Z_{t_{bc}} - 2Z_{t_{bc}}Z_{t_{ca}} & 0 \end{bmatrix}$$

where

$$\begin{aligned}\sum Z_t &= Z_{t_{ab}} + Z_{t_{bc}} + Z_{t_{ca}} \\ [c_t] &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ [d_t] &= \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ [A_t] &= \frac{1}{3n_t} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \\ [B_t] &= \frac{1}{3 \sum Z_t} \begin{bmatrix} 2Z_{t_{ab}}Z_{t_{ca}} + Z_{t_{bc}}Z_{t_{ca}} & -2Z_{t_{ab}}Z_{t_{bc}} + Z_{t_{bc}}(Z_{t_{ab}} + Z_{t_{ca}}) & 0 \\ 2Z_{t_{bc}}Z_{t_{ca}} - Z_{t_{bc}}(Z_{t_{ab}} + Z_{t_{bc}}) & 2Z_{t_{bc}}(Z_{t_{ab}} + Z_{t_{ca}}) - Z_{t_{bc}}Z_{t_{ca}} & 0 \\ Z_{t_{ab}}Z_{t_{ca}} - 2Z_{t_{ca}}(Z_{t_{ab}} + Z_{t_{bc}}) & -Z_{t_{ab}}Z_{t_{bc}} - 2Z_{t_{bc}}Z_{t_{ca}} & 0 \end{bmatrix}\end{aligned}$$

where

$$\sum Z_t = Z_{t_{ab}} + Z_{t_{bc}} + Z_{t_{ca}}$$

21.1.5.5 Thevenin Equivalent Circuit

The study of short-circuit studies that occur on the load side of a transformer bank requires the three-phase Thevenin equivalent circuit referenced to the load side terminals of the transformer. In order to determine this equivalent circuit, the Thevenin equivalent circuit up to the primary terminals of the “feeder” transformer must be determined. It is assumed that the transformer matrices as defined above are known for the transformer connection in question. A one-line diagram of the total system is shown in Fig. 21.22.

The desired Thevenin equivalent circuit on the secondary side of the transformer is shown in Fig. 21.23.

In Fig. 21.22 the system voltage source $[ELN_{ABC}]$ will typically be a balanced set of per-unit voltages. The Thevenin equivalent voltage on the secondary side of the transformer will be:

$$[Eth_{abc}] = [A_t] \cdot [ELN_{ABC}] \quad (21.155)$$

The Thevenin equivalent impedance in Fig. 21.23 from the source to the primary terminals of the feeder transformer is given by

$$[Zth_{abc}] = [A_t][Zsys_{ABC}][d_t] + [B_t] \quad (21.156)$$

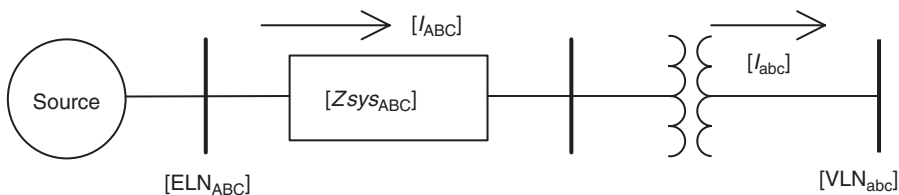


FIGURE 21.22 Total system.

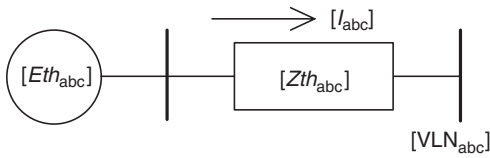


FIGURE 21.23 Three-phase Thevenin equivalent circuit.

The values of the source side Thevenin equivalent circuit will be the same regardless of the type of connection of the feeder transformer. For each three-phase transformer connection, unique values of the matrices $[Eth_{abc}]$ and $[Zth_{abc}]$ are defined as functions of the source side Thevenin equivalent circuit. These definitions are shown for each transformer connection below.

21.1.5.6 Center Tapped Transformers

The typical single-phase service to a customer is 120/240 V. This is provided from a center tapped single-phase transformer through the three-wire secondary and service drop to the customer's meter. The center tapped single-phase transformer with the three-wire secondary and 120/240 V loads can be modeled as shown in Fig. 21.24.

Notice in Fig. 21.24 that three impedance values are required for the center tapped transformer. These three impedances typically will not be known. In fact, usually only the magnitude of the transformer impedance will be known as found on the nameplate. In order to perform a reasonable analysis, some assumptions have to be made regarding the impedances. It is necessary to know both the per-unit R_A and the X_A components of the transformer impedance. References Gonen (1986) and Hopkinson (1977) are two sources for typical values. From Hopkinson (1977) the center tapped transformer impedances as a function of the transformer impedance are given. For interlaced transformers the three impedances are given by

$$\begin{aligned} Z_0 &= 0.5R_A + j0.8X_A \\ Z_1 &= R_A + j0.4X_A \\ Z_2 &= R_A + j0.4X_A \end{aligned} \quad (21.157)$$

The equations for the noninterlaced design are

$$\begin{aligned} Z_0 &= 0.25R_A - j0.6X_A \\ Z_1 &= 1.5R_A + j3.3X_A \\ Z_2 &= 1.5R_A + j3.1X_A \end{aligned} \quad (21.158)$$

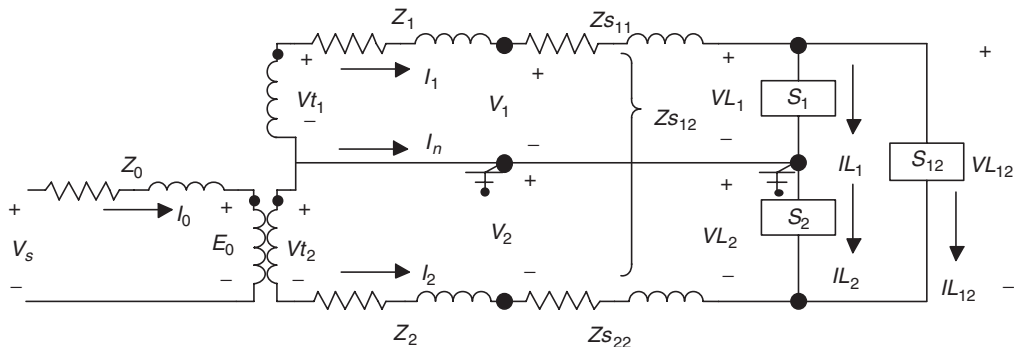


FIGURE 21.24 Center tapped single-phase transformer with secondary.

The transformer turns ratio is defined as

$$n_t = \frac{\text{high side rated voltage}}{\text{low side half winding rated voltage}} \quad (21.159)$$

$$\text{Example: } n_t = \frac{7200}{120} = 60$$

With reference to [Fig. 21.24](#), note that the secondary current I_1 flows out of the dot of the secondary half winding whereas the current I_2 flows out of the undotted terminal. This is done in order to simplify the voltage drop calculations down the secondary. The basic transformer equations that must apply at all times are

$$\begin{aligned} E_0 &= n_t V_{t1} = n_t V_{t2} \\ I_0 &= \frac{1}{n_t} (I_1 - I_2) \end{aligned} \quad (21.160)$$

General matrix equations similar to those of the three-phase transformer connections are used in the analysis. For the backward sweep (working from the load toward the source), the equations are

$$\begin{aligned} [V_{ss}] &= [a_{ct}][V_{12}] + [b_{ct}][I_{12}] \\ [I_{00}] &= [d_{ct}][I_{12}] \end{aligned} \quad (21.161)$$

where

$$\begin{aligned} [V_{ss}] &= \begin{bmatrix} V_s \\ V_s \end{bmatrix} \\ [I_{00}] &= \begin{bmatrix} I_0 \\ I_0 \end{bmatrix} \\ [I_{12}] &= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\ [a_{ct}] &= \begin{bmatrix} n_t & 0 \\ 0 & n_t \end{bmatrix} \\ [b_{ct}] &= \begin{bmatrix} n_t \left(Z_1 + \frac{Z_0}{n_t^2} \right) & -\frac{Z_0}{n_t} \\ \frac{Z_0}{n_t} & -n_t \left(Z_2 + \frac{Z_0}{n_t^2} \right) \end{bmatrix} \\ [d_{ct}] &= \frac{1}{n_t} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \end{aligned} \quad (21.162)$$

For the forward sweep (working from the source toward the loads) the equations are

$$[V_{12}] = [A_{ct}][V_{ss}] - [B_{ct}][I_{12}] \quad (21.163)$$

where

$$\begin{aligned} [A_{ct}] &= \frac{1}{n_t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ [B_{ct}] &= \begin{bmatrix} Z_1 + \frac{Z_0}{n_t^2} & -\frac{Z_0}{n_t^2} \\ \frac{Z_0}{n_t^2} & -\left(Z_2 + \frac{Z_0}{n_t^2} \right) \end{bmatrix} \end{aligned} \quad (21.164)$$

The three-wire secondary is modeled by first applying the Carson's equations and Kron reduction method to determine the 2×2 phase impedance matrix:

$$[Z_s] = \begin{bmatrix} Z_{s_{11}} & Z_{s_{12}} \\ Z_{s_{21}} & Z_{s_{22}} \end{bmatrix} \quad (21.165)$$

The backward sweep equation becomes

$$[V_{12}] = [a_s][V_{L12}] + [b_s][I_{12}]$$

where

$$\begin{aligned} [a_s] &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ [b_s] &= [Z_s] \end{aligned} \quad (21.166)$$

The forward sweep equation is:

$$[V_{L12}] = [A_s][V_{12}] - [B_s][I_{12}] \quad (21.167)$$

where

$$\begin{aligned} [A_s] &= [a_s]^{-1} \\ [B_s] &= [Z_s] \end{aligned} \quad (21.168)$$

21.1.6 Load Models

Loads can be represented as being connected phase-to-phase or phase-to-neutral in a four-wire wye systems or phase-to-phase in a three-wire delta system. The loads can be three-phase, two-phase, or single-phase with any degree of unbalance and can be modeled as

- Constant real and reactive power (constant PQ)
- Constant current
- Constant impedance
- Any combination of the above

The load models developed in this document are used in the iterative process of a power-flow program. All models are initially defined by a complex power per phase and either a line-to-neutral (wye load) or a line-to-line voltage (delta load). The units of the complex power can be in volt-amperes and volts or per-unit volt-amperes and per-unit volts.

For both the wye and delta connected loads, the basic requirement is to determine the load component of the line currents coming into the loads. It is assumed that all loads are initially specified by their complex power ($S = P + jQ$) per phase and a line-to-neutral or line-to-line voltage.

21.1.6.1 Wye Connected Loads

Figure 21.25 shows the model of a wye connected load.

The notation for the specified complex powers and voltages is as follows:

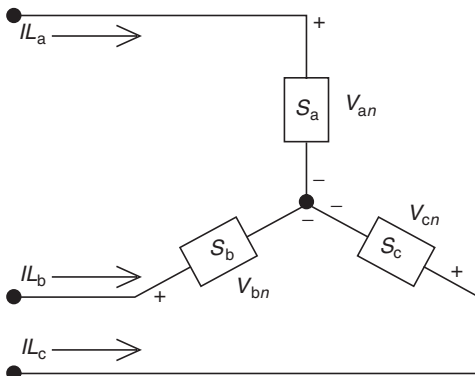


FIGURE 21.25 Wye connected load.

$$\text{Phase a: } |S_a|/\angle\theta_a = P_a + jQ_a \quad \text{and} \quad |V_{an}|/\angle\delta_a \quad (21.169)$$

$$\text{Phase b: } |S_b|/\angle\theta_b = P_b + jQ_b \quad \text{and} \quad |V_{bn}|/\angle\delta_b \quad (21.170)$$

$$\text{Phase c: } |S_c|/\angle\theta_c = P_c + jQ_c \quad \text{and} \quad |V_{cn}|/\angle\delta_c \quad (21.171)$$

1. Constant real and reactive power loads

$$\begin{aligned} IL_a &= \left(\frac{S_a}{V_{an}} \right)^* = \frac{|S_a|}{|V_{an}|} \angle\delta_a - \theta_a = |IL_a|/\angle\alpha_a \\ IL_b &= \left(\frac{S_b}{V_{bn}} \right)^* = \frac{|S_b|}{|V_{bn}|} \angle\delta_b - \theta_b = |IL_b|/\angle\alpha_b \\ IL_c &= \left(\frac{S_c}{V_{cn}} \right)^* = \frac{|S_c|}{|V_{cn}|} \angle\delta_c - \theta_c = |IL_c|/\angle\alpha_c \end{aligned} \quad (21.172)$$

In this model the line-to-neutral voltages will change during each iteration until convergence is achieved.

2. Constant impedance loads

The “constant load impedance” is first determined from the specified complex power and line-to-neutral voltages according to the following equation:

$$\begin{aligned} Z_a &= \frac{|V_{an}|^2}{S_a^*} = \frac{|V_{an}|^2}{|S_a|} \angle\theta_a = |Z_a|/\angle\theta_a \\ Z_b &= \frac{|V_{bn}|^2}{S_b^*} = \frac{|V_{bn}|^2}{|S_b|} \angle\theta_b = |Z_b|/\angle\theta_b \\ Z_c &= \frac{|V_{cn}|^2}{S_c^*} = \frac{|V_{cn}|^2}{|S_c|} \angle\theta_c = |Z_c|/\angle\theta_c \end{aligned} \quad (21.173)$$

The load currents as a function of the constant load impedances are given by the following equation:

$$\begin{aligned} IL_a &= \frac{V_{an}}{Z_a} = \frac{|V_{an}|}{|Z_a|} \angle\delta_a - \theta_a = |IL_a|/\angle\alpha_a \\ IL_b &= \frac{V_{bn}}{Z_b} = \frac{|V_{bn}|}{|Z_b|} \angle\delta_b - \theta_b = |IL_b|/\angle\alpha_b \\ IL_c &= \frac{V_{cn}}{Z_c} = \frac{|V_{cn}|}{|Z_c|} \angle\delta_c - \theta_c = |IL_c|/\angle\alpha_c \end{aligned} \quad (21.174)$$

In this model the line-to-neutral voltages will change during each iteration until convergence is achieved.

3. Constant current loads

In this model the magnitudes of the currents are computed according to Eq. (21.172) and then held constant while the angle of the voltage (δ) changes during each iteration. In order to keep the power factor constant, the angles of the load currents are given by

$$\begin{aligned} IL_a &= |IL_a|/\angle\delta_a - \theta_a \\ IL_b &= |IL_b|/\angle\delta_b - \theta_b \\ IL_c &= |IL_c|/\angle\delta_c - \theta_c \end{aligned} \quad (21.175)$$

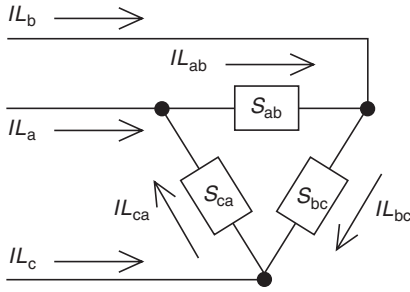


FIGURE 21.26 Delta connected load.

4. Combination loads

Combination loads can be modeled by assigning a percentage of the total load to each of the above three load models. The total line current entering the load is the sum of the three components.

21.1.6.2 Delta Connected Loads

Figure 21.26 shows the model of a delta connected load.

The notation for the specified complex powers and voltages is as follows:

$$\text{Phase ab: } |S_{ab}| \angle \theta_{ab} = P_{ab} + jQ_{ab} \quad \text{and} \quad |V_{ab}| \angle \delta_{ab} \quad (21.176)$$

$$\text{Phase bc: } |S_{bc}| \angle \theta_{bc} = P_{bc} + jQ_{bc} \quad \text{and} \quad |V_{bc}| \angle \delta_{bc} \quad (21.177)$$

$$\text{Phase ca: } |S_{ca}| \angle \theta_{ca} = P_{ca} + jQ_{ca} \quad \text{and} \quad |V_{ca}| \angle \delta_{ca} \quad (21.178)$$

1. Constant real and reactive power loads

$$\begin{aligned} IL_{ab} &= \left(\frac{S_{ab}}{V_{ab}} \right)^* = \frac{|S_{ab}|}{|V_{ab}|} \angle \delta_{ab} - \theta_{ab} = |IL_{ab}| \angle \alpha_{ab} \\ IL_{bc} &= \left(\frac{S_{bc}}{V_{bc}} \right)^* = \frac{|S_{bc}|}{|V_{bc}|} \angle \delta_{bc} - \theta_{bc} = |IL_{bc}| \angle \alpha_{bc} \\ IL_{ca} &= \left(\frac{S_{ca}}{V_{ca}} \right)^* = \frac{|S_{ca}|}{|V_{ca}|} \angle \delta_{ca} - \theta_{ca} = |IL_{ca}| \angle \alpha_{ac} \end{aligned} \quad (21.179)$$

In this model the line-to-line voltages will change during each iteration until convergence is achieved.

2. Constant impedance loads

The constant load impedance is first determined from the specified complex power and line-to-neutral voltages according to the following equation:

$$\begin{aligned} Z_{ab} &= \frac{|V_{ab}|^2}{S_{ab}^*} = \frac{|V_{ab}|^2}{|S_{ab}|} \angle \theta_{ab} = |Z_{ab}| \angle \theta_{ab} \\ Z_{bc} &= \frac{|V_{bc}|^2}{S_{bc}^*} = \frac{|V_{bc}|^2}{|S_{bc}|} \angle \theta_{bc} = |Z_{bc}| \angle \theta_{bc} \\ Z_{ca} &= \frac{|V_{ca}|^2}{S_{ca}^*} = \frac{|V_{ca}|^2}{|S_{ca}|} \angle \theta_{ca} = |Z_{ca}| \angle \theta_{ca} \end{aligned} \quad (21.180)$$

The load currents as a function of the constant load impedances are given by the following equation:

$$\begin{aligned} IL_{ab} &= \frac{V_{ab}}{Z_{ab}} = \frac{|V_{ab}|}{|Z_{ab}|} \angle \delta_{ab} - \theta_{ab} = |IL_{ab}| \angle \alpha_{ab} \\ IL_{bc} &= \frac{V_{bc}}{Z_{bc}} = \frac{|V_{bc}|}{|Z_{bc}|} \angle \delta_{bc} - \theta_{bc} = |IL_{bc}| \angle \alpha_{bc} \\ IL_{ca} &= \frac{V_{ca}}{Z_{ca}} = \frac{|V_{ca}|}{|Z_{ca}|} \angle \delta_{ca} - \theta_{ca} = |IL_{ca}| \angle \alpha_{ca} \end{aligned} \quad (21.181)$$

In this model the line-to-line voltages in Eq. (21.181) will change during each iteration until convergence is achieved.

3. Constant current loads

In this model the magnitudes of the currents are computed according to Eq. (21.179) and then held constant while the angle of the voltage (δ) changes during each iteration. This keeps the power factor of the load constant.

$$\begin{aligned} IL_{ab} &= |IL_{ab}| \angle \delta_{ab} - \theta_{ab} \\ IL_{bc} &= |IL_{bc}| \angle \delta_{bc} - \theta_{bc} \\ IL_{ca} &= |IL_{ca}| \angle \delta_{ca} - \theta_{ca} \end{aligned} \quad (21.182)$$

4. Combination loads

Combination loads can be modeled by assigning a percentage of the total load to each of the above three load models. The total delta current for each load is the sum of the three components.

The line currents entering the delta connected load for all models are determined by

$$\begin{bmatrix} IL_a \\ IL_b \\ IL_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} IL_{ab} \\ IL_{bc} \\ IL_{ca} \end{bmatrix} \quad (21.183)$$

In both the wye and delta connected loads, single-phase and two-phase loads are modeled by setting the complex powers of the missing phases to zero. In other words, all loads are modeled as three-phase loads and by setting the complex power of the missing phases to zero, the only load currents computed using the above equations will be for the nonzero loads.

21.1.7 Shunt Capacitor Models

Shunt capacitor banks are commonly used in a distribution system to help in voltage regulation and to provide reactive power support. The capacitor banks are modeled as constant susceptances connected in either wye or delta. Similar to the load model, all capacitor banks are modeled as three-phase banks with the kVAr of missing phases set to zero for single-phase and two-phase banks.

21.1.7.1 Wye Connected Capacitor Bank

A wye connected capacitor bank is shown in Fig. 21.27. The individual phase capacitor units are specified in kVAr and kV. The constant susceptance for each unit can be computed in either Siemens or per unit. When per unit is desired, the specified kVAr of the capacitor must be divided by the base single-phase kVAr and the kV must be divided by the base line-to-neutral kV.

The susceptance of a capacitor unit is computed by

$$B_{\text{actual}} = \frac{\text{kVAr}}{\text{kV}^2 1000} \quad \text{Siemens} \quad (21.184)$$

$$B_{\text{pu}} = \frac{\text{kVAr}_{\text{pu}}}{V_{\text{pu}}^2} \quad \text{per unit} \quad (21.185)$$

$$\text{where } \text{kVAr}_{\text{pu}} = \frac{\text{kVAr}_{\text{actual}}}{\text{kVA}_{\text{single_phase_base}}} \quad (21.186)$$

$$V_{\text{pu}} = \frac{\text{kV}_{\text{actual}}}{\text{kV}_{\text{line_to_neutral_base}}} \quad (21.187)$$

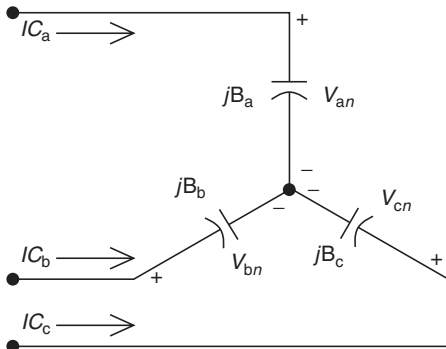


FIGURE 21.27 Wye connected capacitor bank.

The per-unit value of the susceptance can also be determined by first computing the actual value [Eq. (21.184)] and then dividing by the base admittance of the system.

With the susceptance computed, the line currents serving the capacitor bank are given by

$$\begin{aligned} I_{C_a} &= jB_a V_{an} \\ I_{C_b} &= jB_b V_{bn} \\ I_{C_c} &= jB_c V_{cn} \end{aligned} \quad (21.188)$$

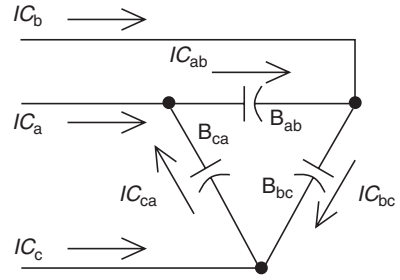


FIGURE 21.28 Delta connected capacitor bank.

21.1.7.2 Delta Connected Capacitor Bank

A delta connected capacitor bank is shown in Fig. 21.28.

Equations (21.184) through (21.187) can be used to determine the value of the susceptance in actual Siemens or per unit. It should be pointed out that in this case, the kV will be a line-to-line value of the voltage. Also, it should be noted that in Eq. (21.187), the base line-to-neutral voltage is used to compute the per-unit line-to-line voltage. This is a variation from the usual application of the per-unit system where the actual line-to-line voltage would be divided by a base line-to-line voltage in order to get the per-unit line-to-line voltage. That is not done here so that under normal conditions, the per-unit line-to-line voltage will have a magnitude of $\sqrt{3}$ rather than 1.0. This is done so that Kirchhoff's current law (KCL) at each node of the delta connection will apply for either the actual or per-unit delta currents. The currents flowing in the delta connected capacitors are given by

$$\begin{aligned} I_{C_{ab}} &= jB_{ab} V_{ab} \\ I_{C_{bc}} &= jB_{bc} V_{bc} \\ I_{C_{ca}} &= jB_{ca} V_{ca} \end{aligned} \quad (21.189)$$

The line currents feeding the delta connected capacitor bank are given by

$$\begin{bmatrix} I_{C_a} \\ I_{C_b} \\ I_{C_c} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{C_{ab}} \\ I_{C_{bc}} \\ I_{C_{ca}} \end{bmatrix} \quad (21.190)$$

21.2 Analysis

21.2.1 Power-Flow Analysis

The power-flow analysis of a distribution feeder is similar to that of an interconnected transmission system. Typically what will be known prior to the analysis will be the three-phase voltages at the substation and the complex power of all the loads and the load model (constant complex power, constant impedance, constant current, or a combination). Sometimes, the input complex power supplied to the feeder from the substation is also known.

In Sections 21.1.3, 21.1.4, and 21.1.5, phase frame models were presented for the series components of a distribution feeder. In Sections 21.1.6 and 21.1.7, models were presented for the shunt components (loads and capacitor banks). These models are used in the “power-flow” analysis of a distribution feeder.

A power-flow analysis of a feeder can determine the following by phase and total three-phase:

- Voltage magnitudes and angles at all nodes of the feeder
- Line flow in each line section specified in kW and kVAr, amps and degrees, or amps and power factor

- Power loss in each line section
- Total feeder input kW and kVAr
- Total feeder power losses
- Load kW and kVAr based upon the specified model for the load

Because the feeder is radial, iterative techniques commonly used in transmission network power-flow studies are not used because of poor convergence characteristics (Trevino, 1970). Instead, an iterative technique specifically designed for a radial system is used. The ladder iterative technique (Kersting and Mendive, 1976) will be presented here.

21.2.1.1 The Ladder Iterative Technique

21.2.1.1.1 Linear Network

A modification of the ladder network theory of linear systems provides a robust iterative technique for power-flow analysis. A distribution feeder is nonlinear because most loads are assumed to be constant kW and kVAr. However, the approach taken for the linear system can be modified to take into account the nonlinear characteristics of the distribution feeder.

For the ladder network in Fig. 21.29, it is assumed that all of the line impedances and load impedances are known along with the voltage at the source (V_s).

The solution for this network is to assume a voltage at the most remote load (V_5). The load current I_5 is then determined as

$$I_5 = \frac{V_5}{ZL_5} \quad (21.191)$$

For this “end-node” case, the line current I_{45} is equal to the load current I_5 . The voltage at node 4 (V_4) can be determined using Kirchhoff’s voltage law (KVL):

$$V_4 = V_5 + Z_{45}I_{45} \quad (21.192)$$

The load current I_4 can be determined and then KCL applied to determine the line current I_{34} .

$$I_{34} = I_{45} + I_4 \quad (21.193)$$

KVL is applied to determine the node voltage V_3 . This procedure is continued until a voltage (V_1) has been computed at the source. The computed voltage V_1 is compared to the specified voltage V_s . There will be a difference between these two voltages. The ratio of the specified voltage to the compute voltage can be determined as

$$\text{Ratio} = \frac{V_s}{V_1} \quad (21.194)$$

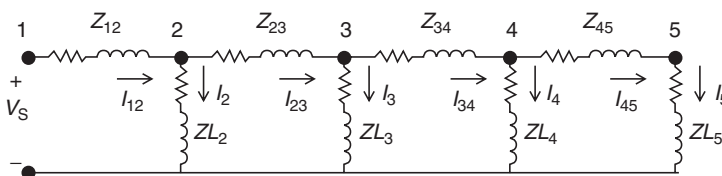


FIGURE 21.29 Linear ladder network.

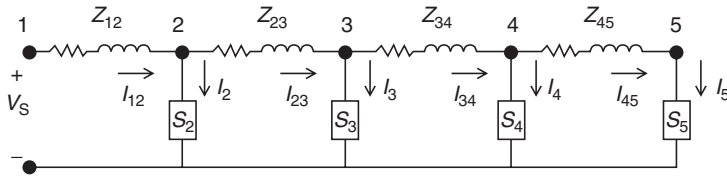


FIGURE 21.30 Nonlinear ladder network.

Since the network is linear, all of the line and load currents and node voltages in the network can be multiplied by the Ratio for the final solution to the network.

21.2.1.1.2 Nonlinear Network

The linear network of Fig. 21.29 is modified to a nonlinear network by replacing all of the constant load impedances by constant complex power loads as shown in Fig. 21.30.

The procedure outlined for the linear network is applied initially to the nonlinear network. The only difference being that the load current (assuming constant P and Q) at each node is computed by

$$I_n = \left(\frac{S_n}{V_n} \right)^* \quad (21.195)$$

The backward sweep will determine a computed source voltage V_1 . As in the linear case, this first iteration will produce a voltage that is not equal to the specified source voltage V_s . Because the network is nonlinear, multiplying currents and voltages by the ratio of the specified voltage to the computed voltage will not give the solution. The most direct modification to the ladder network theory is to perform a forward sweep. The forward d sweep commences by using the specified source voltage and the line currents from the backward sweep. KVL is used to compute the voltage at node 2 by

$$V_2 = V_s - Z_{12}I_{12} \quad (21.196)$$

This procedure is repeated for each line segment until a “new” voltage is determined at node 5. Using the new voltage at node 5, a second backward sweep is started that will lead to a new computed voltage at the source. The backward and forward sweep process is continued until the difference between the computed and specified voltage at the source is within a given tolerance.

21.2.1.1.3 General Feeder

A typical distribution feeder will consist of the “primary main” with laterals tapped off the primary main, and sublaterals tapped off the laterals, etc. Figure 21.30 shows an example of a typical feeder.

The ladder iterative technique for the feeder of Fig. 21.31 would proceed as follows:

1. Assume voltages (1.0 per unit) at the “end” nodes (6, 8, 9, 11, and 13).
2. Starting at node 13, compute the node current (load current plus capacitor current if present).
3. With this current, apply KVL to calculate the node voltages at 12 and 10.
4. Node 10 is referred to as a “junction” node since laterals branch in two directions from the node. This feeder goes to node 11 and computes the node current. Use that current to compute the voltage at node 10. This will be referred to as “the most recent voltage at node 10.”
5. Using the most recent value of the voltage at node 10, the node current at node 10 (if any) is computed.
6. Apply KCL to determine the current flowing from node 4 toward node 10.
7. Compute the voltage at node 4.

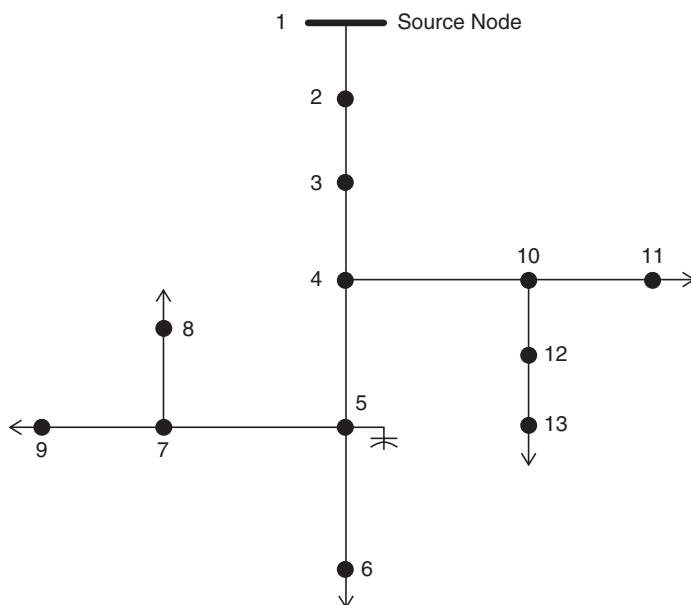


FIGURE 21.31 Typical distribution feeder.

8. Node 4 is a junction node. An end-node downstream from node 4 is selected to start the forward sweep toward node 4.
9. Select node 6, compute the node current, and then compute the voltage at junction-node 5.
10. Go to downstream end-node 8. Compute the node current and then the voltage at junction-node 7.
11. Go to downstream end-node 9. Compute the node current and then the voltage at junction-node 7.
12. Compute the node current at node 7 using the most recent value of node 7 voltage.
13. Apply KCL at node 7 to compute the current flowing on the line segment from node 5 to node 7.
14. Compute the voltage at node 5.
15. Compute the node current at node 5.
16. Apply KCL at node 5 to determine the current flowing from node 4 toward node 5.
17. Compute the voltage at node 4.
18. Compute the node current at node 4.
19. Apply KCL at node 4 to compute the current flowing from node 3 to node 4.
20. Calculate the voltage at node 3.
21. Compute the node current at node 3.
22. Apply KCL at node 3 to compute the current flowing from node 2 to node 3.
23. Calculate the voltage at node 2.
24. Compute the node current at node 2.
25. Apply KCL at node 2.
26. Calculate the voltage at node 1.
27. Compare the calculated voltage at node 1 to the specified source voltage.
28. If not within tolerance, use the specified source voltage and the backward sweep current flowing from node 1 to node 2 and compute the new voltage at node 2.
29. The forward sweep continues using the new upstream voltage and line segment current from the forward sweep to compute the new downstream voltage.
30. The forward sweep is completed when new voltages at all end nodes have been completed.
31. This completes the first iteration.
32. Now repeat the backward sweep using the new end voltages rather than the assumed voltages as was done in the first iteration.

33. Continue the backward and forward sweeps until the calculated voltage at the source is within a specified tolerance of the source voltage.
34. At this point, the voltages are known at all nodes and the currents flowing in all line segments are known. An output report can be produced giving all desired results.

21.2.1.2 The Unbalanced Three-Phase Distribution Feeder

The previous section outlined the general procedure for performing the ladder iterative technique. This section will address how that procedure can be used for an unbalanced three-phase feeder.

Figure 21.32 is the one-line diagram of an unbalanced three-phase feeder. The topology of the feeder in Fig. 21.32 is the same as the feeder in Fig. 21.31. Figure 21.32 shows more detail of the feeder however. The feeder in Fig. 21.32 can be broken into the series components and the shunt components.

21.2.1.2.1 Series Components

The series components of a distribution feeder are

- Line segments
- Transformers
- Voltage regulators

Models for each of the series components have been developed in prior areas of this section. In all cases, models (three-phase, two-phase, and single-phase) were developed in such a manner that they can be generalized. Figure 21.33 shows the “general model” for each of the series components.

The general equations defining the “input” (node n) and “output” (node m) voltages and currents are given by

$$[V_{abc}]_n = [a][V_{abc}]_m + [b][I_{abc}]_m \quad (21.197)$$

$$[I_{abc}]_n = [c][V_{abc}]_m + [d][I_{abc}]_m \quad (21.198)$$

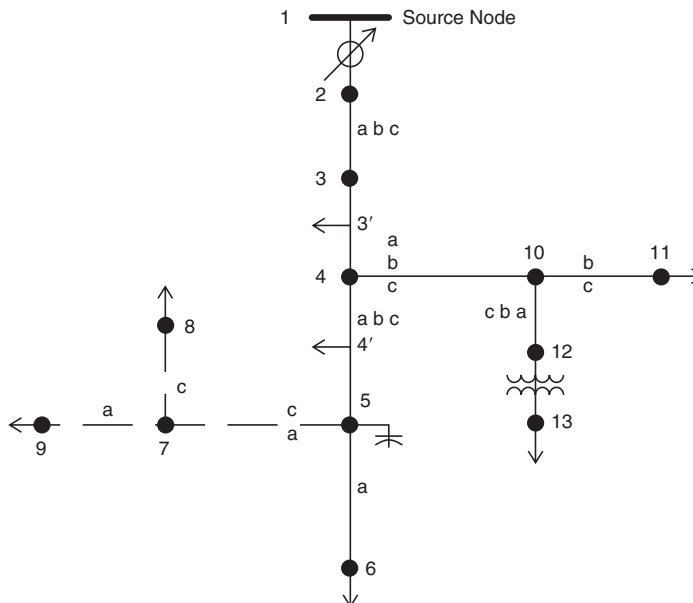


FIGURE 21.32 Unbalanced three-phase distribution feeder.



FIGURE 21.33 Series feeder component.

The general equation relating the output (node m) and input (node n) voltages is given by

$$[V_{abc}]_m = [A][V_{abc}]_n - [B][I_{abc}]_m \quad (21.199)$$

In Eqs. (21.197) through (21.199), the voltages are line-to-neutral for a four-wire wye feeder and equivalent line-to-neutral for a three-wire delta system. For transformers and voltage regulators, the voltages are line-to-neutral for terminals that are connected to a four-wire wye and line-to-line when connected to a three-wire delta.

21.2.1.2.2 Shunt Components

The shunt components of a distribution feeder are

- Spot loads
- Distributed loads
- Capacitor banks

Spot loads are located at a node and can be three-phase, two-phase, or single-phase and connected in either a wye or a delta connection. The loads can be modeled as constant complex power, constant current, constant impedance, or a combination of the three.

Distributed loads are located at the midsection of a line segment. A distributed load is modeled when the loads on a line segment are uniformly distributed along the length of the segment. As in the spot load, the distributed load can be three-phase, two-phase, or single-phase and connected in either a wye or a delta connection. The loads can be modeled as constant complex power, constant current, constant impedance, or a combination of the three. To model the distributed load, a “dummy” node is created in the center of a line segment with the distributed load of the line section modeled at this dummy node.

Capacitor banks are located at a node and can be three-phase, two-phase, or single-phase and can be connected in a wye or delta. Capacitor banks are modeled as constant admittances.

In Fig. 21.32 the solid line segments represent overhead lines while the dashed lines represent underground lines. Note that the phasing is shown for all of the line segments. In the area of the section entitled “Line Impedances,” the application of Carson’s equations for computing the line impedances for overhead and underground lines was presented. There it was pointed out that two-phase and single-phase lines are represented by a 3×3 matrix with zeros set in the rows and columns of the missing phases.

In the area of the section entitled “Line Admittances,” the method for the computation of the shunt capacitive susceptance for overhead and underground lines was presented. Most of the time the shunt capacitance of the line segment can be ignored; however, for long underground segments, the shunt capacitance should be included.

The “node” currents may be three-phase, two-phase, or single-phase and consist of the sum of the load current at the node plus the capacitor current (if any) at the node.

21.2.1.3 Applying the Ladder Iterative Technique

The previous section outlined the steps required for the application of the ladder iterative technique. For the general feeder of Fig. 21.32 the same outline applies. The only difference is that Eq. (21.197) and (21.198) are used for computing the node voltages on the backward sweep and Eq. (21.199) is used for

computing the downstream voltages on the forward sweep. The [a], [b], [c], [d], [A], and [B] matrices for the various series components are defined in the following areas of this section:

- Line segments: Line segment models
- Voltage regulators: Step-voltage regulators
- Transformer banks: Transformer bank connections

The node currents are defined in the following area:

- Loads: Load models
- Capacitors: Shunt capacitor models

21.2.1.4 Final Notes

21.2.1.4.1 Line Segment Impedances

It is extremely important that the impedances and admittances of the line segments be computed using the exact spacings and phasing. Because of the unbalanced loading and resulting unbalanced line currents, the voltage drops due to the mutual coupling of the lines become very important. It is not unusual to observe a voltage rise on a lightly loaded phase of a line segment that has an extreme current unbalance.

21.2.1.4.2 Power Loss

The real power losses of a line segment must be computed as the difference (by phase) of the input power to a line segment minus the output power of the line segment. It is possible to observe a negative power loss on a phase that is lightly loaded compared to the other two phases. Computing power loss as the phase current squared times the phase resistance does not give the actual real power loss in the phases.

21.2.1.4.3 Load Allocation

Many times the input complex power (kW and kVAr) to a feeder is known because of the metering at the substation. This information can be either total three-phase or for each individual phase. In some cases the metered data may be the current and power factor in each phase.

It is desirable to have the computed input to the feeder match the metered input. This can be accomplished (following a converged iterative solution) by computing the ratio of the metered input to the computed input. The phase loads can now be modified by multiplying the loads by this ratio. Because the losses of the feeder will change when the loads are changed, it is necessary to go through the ladder iterative process to determine a new computed input to the feeder. This new computed input will be closer to the metered input, but most likely not within a specified tolerance. Again, a ratio can be determined and the loads modified. This process is repeated until the computed input is within a specified tolerance of the metered input.

21.2.1.5 Short-Circuit Analysis

The computation of short-circuit currents for unbalanced faults in a normally balanced three-phase system has traditionally been accomplished by the application of symmetrical components. However, this method is not well-suited to a distribution feeder that is inherently unbalanced. The unequal mutual coupling between phases leads to mutual coupling between sequence networks. When this happens, there is no advantage to using symmetrical components. Another reason for not using symmetrical components is that the phases between which faults occur is limited. For example, using symmetrical components, line-to-ground faults are limited to phase a to ground. What happens if a single-phase lateral is connected to phase b or c? This section will present a method for short-circuit analysis of an unbalanced three-phase distribution feeder using the phase frame (Kersting, 1980).

21.2.1.5.1 General Theory

[Figure 21.34](#) shows the unbalanced feeder as modeled for short-circuit calculations. In Fig. 21.34, the voltage sources E_a , E_b , and E_c represent the Thevenin equivalent line-to-ground voltages at the faulted

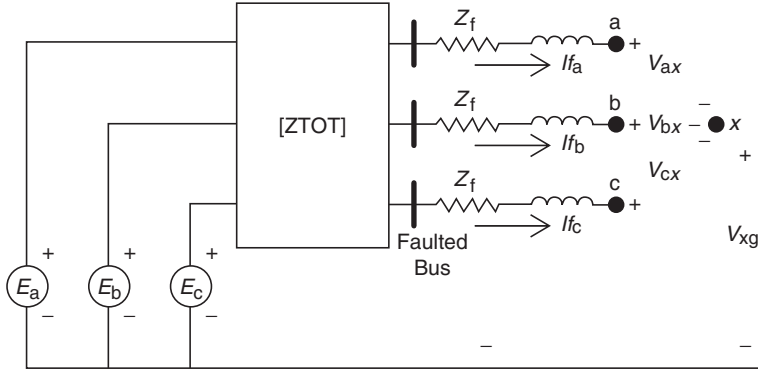


FIGURE 21.34 Unbalanced feeder short-circuit analysis model.

bus. The matrix $[ZTOT]$ represents the Thevenin equivalent impedance matrix at the faulted bus. The fault impedance is represented by Z_f in Fig. 21.34.

Kirchhoff's voltage law in matrix form can be applied to the circuit of Fig. 21.33.

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} + \begin{bmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix} \begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} + \begin{bmatrix} V_{ax} \\ V_{bx} \\ V_{cx} \end{bmatrix} + \begin{bmatrix} V_{xg} \\ V_{xg} \\ V_{xg} \end{bmatrix} \quad (21.200)$$

Equation (21.188) can be written in compressed form as

$$[E_{abc}] = [ZTOT][I_{fabc}] + [ZF][I_{fabc}] + [V_{abcx}] + [V_{xg}] \quad (21.201)$$

Combine terms in Eq. (21.201).

$$[E_{abc}] = [ZEQ][I_{fabc}] + [V_{abcx}] + [V_{xg}] \quad (21.202)$$

$$\text{where } [ZEQ] = [ZTOT] + [ZF] \quad (21.203)$$

Solve Eq. (21.202) for the fault currents:

$$[I_{fabc}] = [YEQ][E_{abc}] - [YEQ][V_{abcx}] - [YEQ][V_{xg}] \quad (21.204)$$

$$\text{where } [YEQ] = [ZEQ]^{-1} \quad (21.205)$$

Since the matrices $[YEQ]$ and $[E_{abc}]$ are known, define

$$[IP_{abc}] = [YEQ][E_{abc}] \quad (21.206)$$

Substituting Eq. (21.206) into Eq. (21.204) results in the following expanded equation:

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} IP_a \\ IP_b \\ IP_c \end{bmatrix} - \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix} \begin{bmatrix} V_{ax} \\ V_{bx} \\ V_{cx} \end{bmatrix} - \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix} \begin{bmatrix} V_{xg} \\ V_{xg} \\ V_{xg} \end{bmatrix} \quad (21.207)$$

Performing the matrix operations in Eq. (21.195):

$$\begin{aligned} If_a &= IP_a - (Y_{aa} V_{ax} + Y_{ab} V_{bx} + Y_{ac} V_{cx}) - Y_a V_{xg} \\ If_b &= IP_b - (Y_{ba} V_{ax} + Y_{bb} V_{bx} + Y_{bc} V_{cx}) - Y_b V_{xg} \\ If_c &= IP_c - (Y_{ca} V_{ax} + Y_{cb} V_{bx} + Y_{cc} V_{cx}) - Y_c V_{xg} \end{aligned} \quad (21.208)$$

where

$$\begin{aligned} Y_a &= Y_{aa} + Y_{ab} + Y_{ac} \\ Y_b &= Y_{ba} + Y_{bb} + Y_{bc} \\ Y_c &= Y_{ca} + Y_{cb} + Y_{cc} \end{aligned} \quad (21.209)$$

Equations (21.208) become the general equations that are used to simulate all types of short circuits. Basically there are three equations and seven unknowns (If_a , If_b , If_c , V_{ax} , V_{bx} , V_{cx} , and V_{xg}). The other three variables in the equations (IP_a , IP_b , and IP_c) are functions of the total impedance and the Thevenin voltages and are therefore known. In order to solve Eq. (21.208), it will be necessary to specify four of the seven unknowns. These specifications are functions of the type of fault being simulated. The additional required four knowns for various types of faults are given below:

Three-phase faults

$$\begin{aligned} V_{ax} &= V_{bx} = V_{cx} = 0 \\ I_a + I_b + I_c &= 0 \end{aligned} \quad (21.210)$$

Three-phase-to-ground faults

$$V_{ax} = V_{bx} = V_{cx} = V_{xg} = 0 \quad (21.211)$$

Line-to-line faults (assume i-j fault with phase k unfaulted)

$$\begin{aligned} V_{ix} &= V_{jx} = 0 \\ If_k &= 0 \\ If_i + If_j &= 0 \end{aligned} \quad (21.212)$$

Line-to-line-to-ground faults (assume i-j to ground fault with k unfaulted)

$$\begin{aligned} V_{ix} &= V_{jx} = V_{xg} = 0 \\ V_{kx} &= \frac{IP_k}{Y_{kk}} \end{aligned} \quad (21.213)$$

Line-to-ground faults (assume phase k fault with phases i and j unfaulted)

$$\begin{aligned} V_{kx} &= V_{xg} = 0 \\ If_i &= If_j = 0 \end{aligned} \quad (21.214)$$

Notice that Eqs. (21.212) through (21.214) will allow the simulation of line-to-line, line-to-line-to-ground, and line-to-ground faults for all phases. There is no limitation to b-c faults for line-to-line and a-g for line-to-ground as is the case when the method of symmetrical components is employed.

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